EECS 598: Convex Optimization Methods in Control

Instructor: Pete Seiler Credits: 3 Pre-requisite: EECS 560 (AERO 550) (ME 564) Linear Systems Theory or instructor permission. Meets: Tu/Th 12:00PM - 1:30PM in 1690 BEYSTER

Course Description

Convex optimization plays a central role in the numerical solution of many design and analysis problems in control theory. This course focuses on the practical aspects of using convex optimization methods to solve these problems. First, the basic properties and theory of convex optimization will be introduced. This will include techniques to formulate engineering problems as convex optimizations. Existing software and implementation issues will also be discussed. Next, we will explore the ties between convex optimization and problems in control theory. This will methods to assess robustness of systems with nonlinearities, parametric uncertainty and unmodeled dynamics. It will also include methods to solve optimal control problems, e.g. H-infinity design. The course will conclude with more recent results in nonlinear analysis, e.g. sumof-squares programming. Example problems of industrial relevance will be used to highlight the utility of these methods.

Course Topics

Introduction 1: Course Overview

Essentials of Convex Optimization

- 2: Convex Sets
- 3: Convex Functions
- 4: Convex Optimization Problems
- 5: Numerical Tools
- 6: Duality
- 7: Unconstrained Minimization
- 8: Constrained Minimization

Nominal Analysis & Synthesis

9: Internal Stability

- 10: State Feedback Synthesis
- 11: Bounded Real Lemma
- 12: H-infinity State Feedback Synthesis
- 13: H-infinity Optimal Control
- 14: H2 Optimal Control
- 15: Linear Parameter Varying Systems

Robust Analysis and Synthesis

- 16: Modeling Uncertain Systems
- 17: Circle Criterion
- 18: Analysis with Unmodeled (Dynamic) Uncertainty
- 19: Analysis with Dynamic LTI Uncertainty
- 20: Pointwise Frequency Domain Constraints
- 21: Analysis with General Integral Quadratic Constraints
- 22: Application to analyzing convergence rates for first-order optimization methods

Nonlinear Analysis with Sum-of-Squares

- 22: Multivariable Polynomials
- 23: Multivariable Polynomials
- 24: Sum-of-Squares Optimization
- 25: Sum-of-Squares Toolbox
- 26: Nonlinear Analysis with SOS
- 27: Nonlinear Analysis with SOS

<u>Miscellaneous Topics (Time Permitting)</u> Youla Parameterization and Optimization with Ritz Approximation Model Reduction Nonconvex QCQPs

References

1. *Convex Optimization* by Boyd and Vandenberghe (http://www.stanford.edu/~boyd/cvxbook/)

2. *Linear Matrix Inequalities in System and Control Theory* by Boyd, El Ghaoui, Feron, and Balakrishnan (<u>http://www.stanford.edu/~boyd/lmibook/</u>)

3. "System analysis via integral quadratic constraints," Megretski and Rantzer, IEEE Transactions on Automatic Control, Vol. 42, # 6, June 1997, pp. 819-830.

4. Structured Semidefinite Programs and Semialgebraic Geometry Methods in Robustness and Optimization by Parrilo (Ph.D. Thesis, California Institute of Technology May 2000). (http://www.mit.edu/~parrilo/pubs/index.html)

5. *Linear Controller Design – Limits of Performance* by Boyd and Barratt (<u>http://www.stanford.edu/~boyd/lcdbook/</u>)