Spatially Coupled LDPC Codes: Is This What Shannon Had In Mind?

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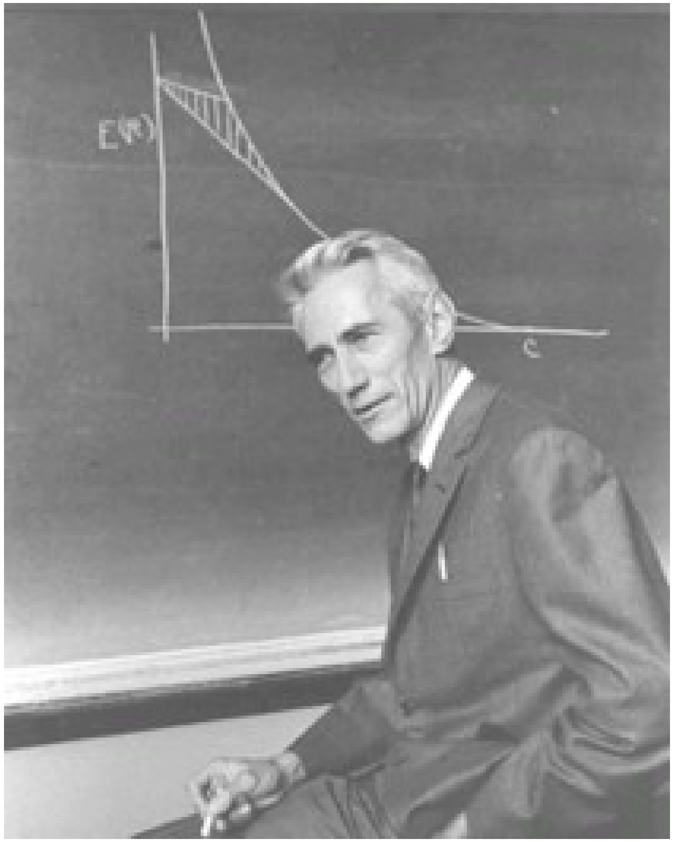
University of Michigan, Nov. 17, 2016

Research Collaborators: David Mitchell, Michael Lentmaier, and Ali Pusane

Outline

- **From Shannon to Modern Coding Theory**
  - ➡ Channel capacity, structured codes, random codes, LDPC codes
- **LDPC Block Codes**
  - ➡ Parity-check matrix and Tanner graph representations, minimum distance bounds, iterative decoding thresholds, protograph-based constructions
- **Spatially Coupled LDPC Codes**
  - ➡ Protograph representation, edge-spreading construction, termination
  - ➡ Iterative decoding thresholds, threshold saturation, minimum distance
- **Practical Considerations**
  - ➡ Window decoding, performance, latency, and complexity comparisons to LDPC block codes, rate-compatibility, implementation aspects

Shannon's Legacy



Claude Elwood Shannon
Apr. 30, 1916 – Feb. 24, 2001
Father of **Information Theory**

Shannon's Legacy

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Shannon's Theory Was Invented at Bell Labs

Bell Labs in Murray Hill,
New Jersey

Shannon's Legacy

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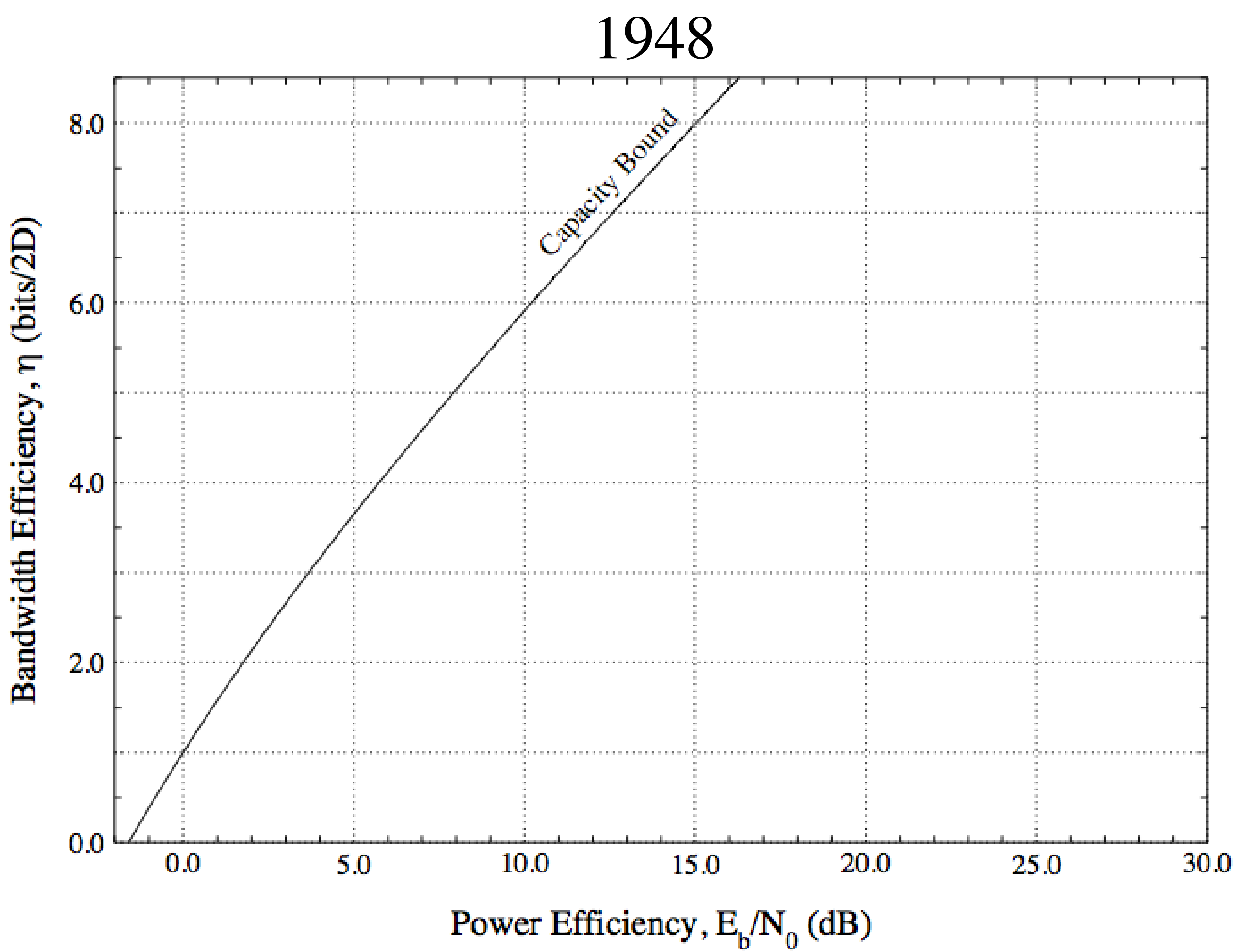
Three Great Successes of Information Theory

- Source Coding for Data Compression
- Secret Coding (Cryptography) for Data Security
- Channel Coding for Data Reliability (*the focus of this presentation*)

Shannon's Legacy



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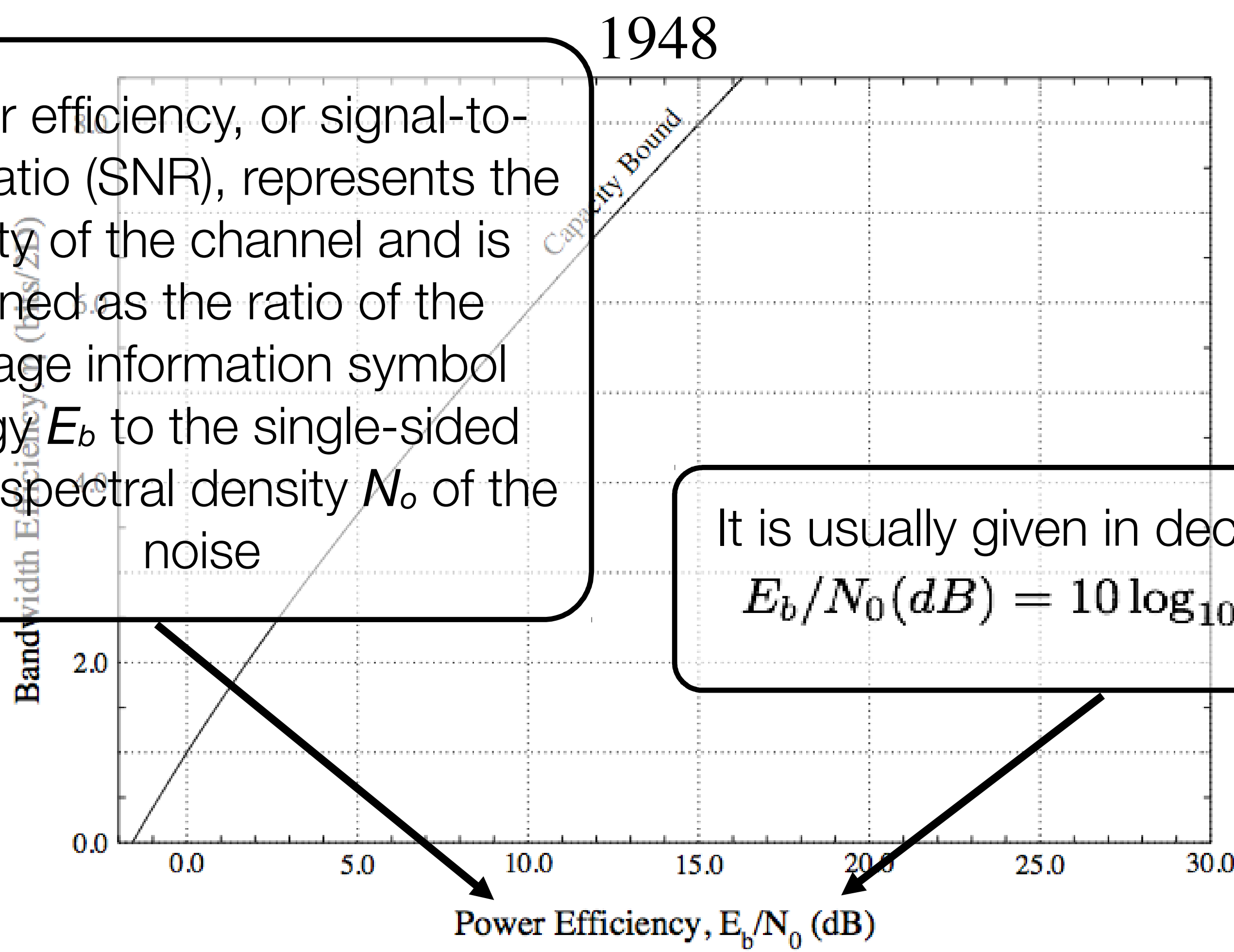
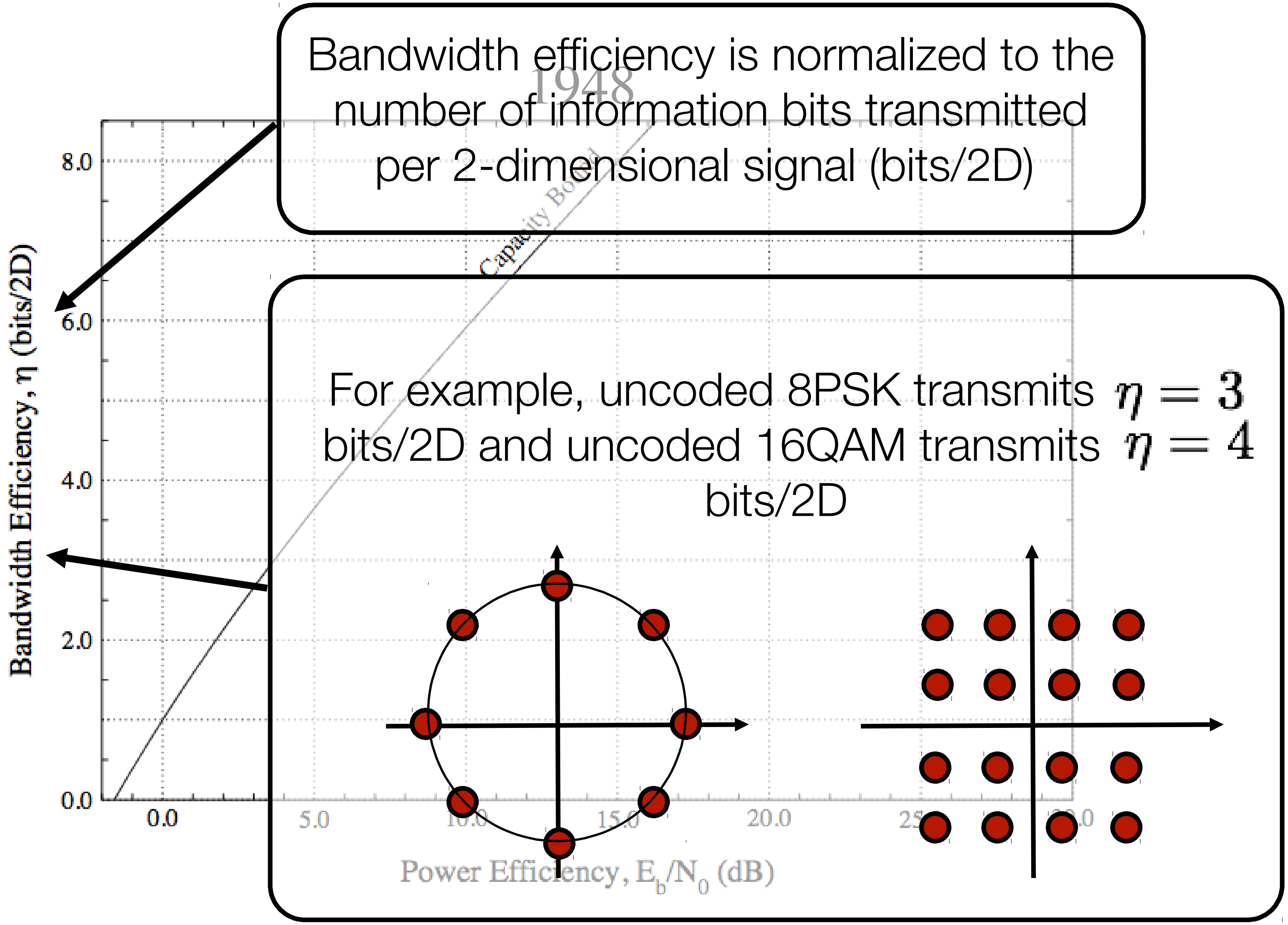


Power efficiency, or signal-to-noise ratio (SNR), represents the quality of the channel and is defined as the ratio of the average information symbol energy E_b to the single-sided power spectral density N_o of the noise

It is usually given in decibels (dB).

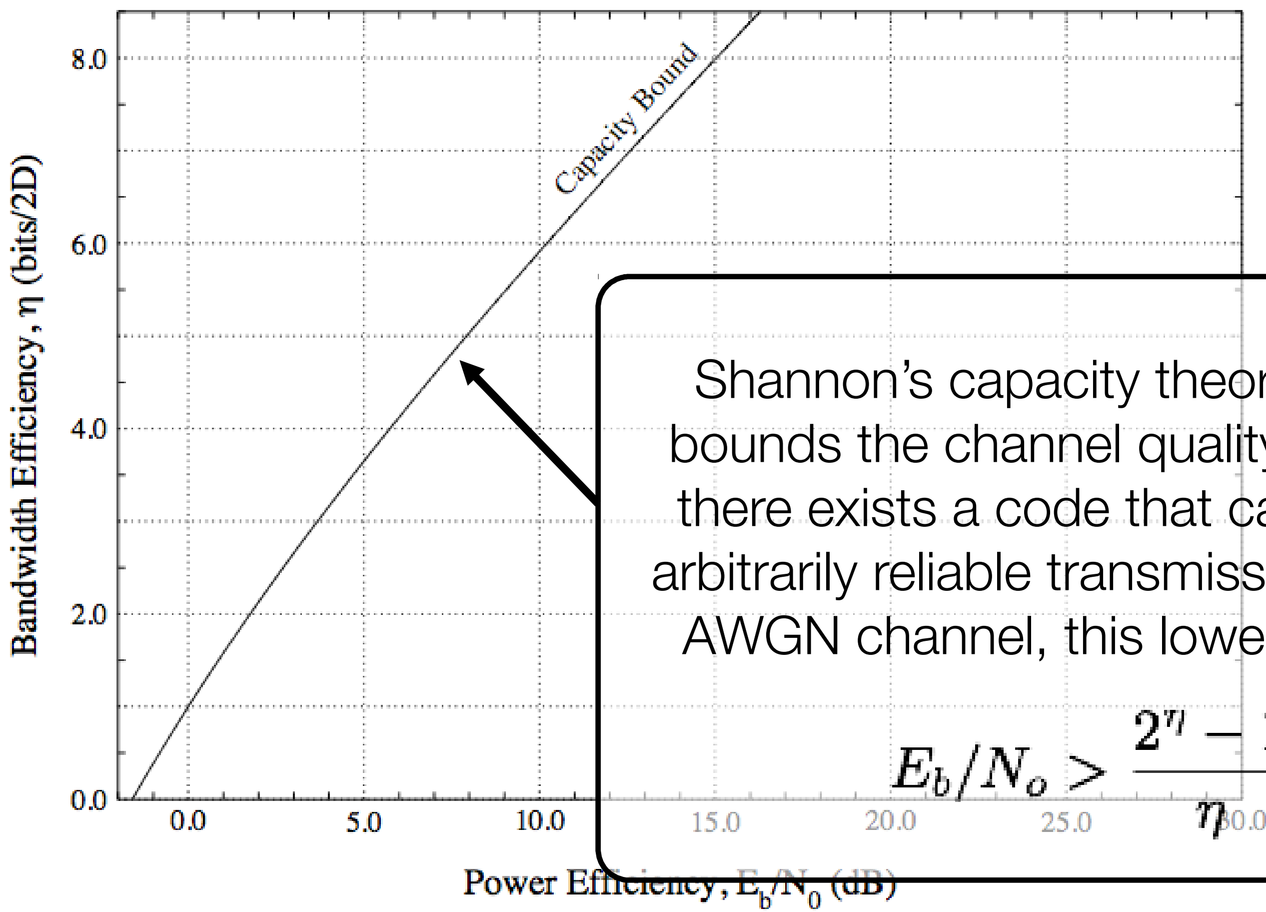
$$E_b/N_0(dB) = 10\log_{10} E_b/N_o$$

Shannon's Legacy



Shannon's Legacy

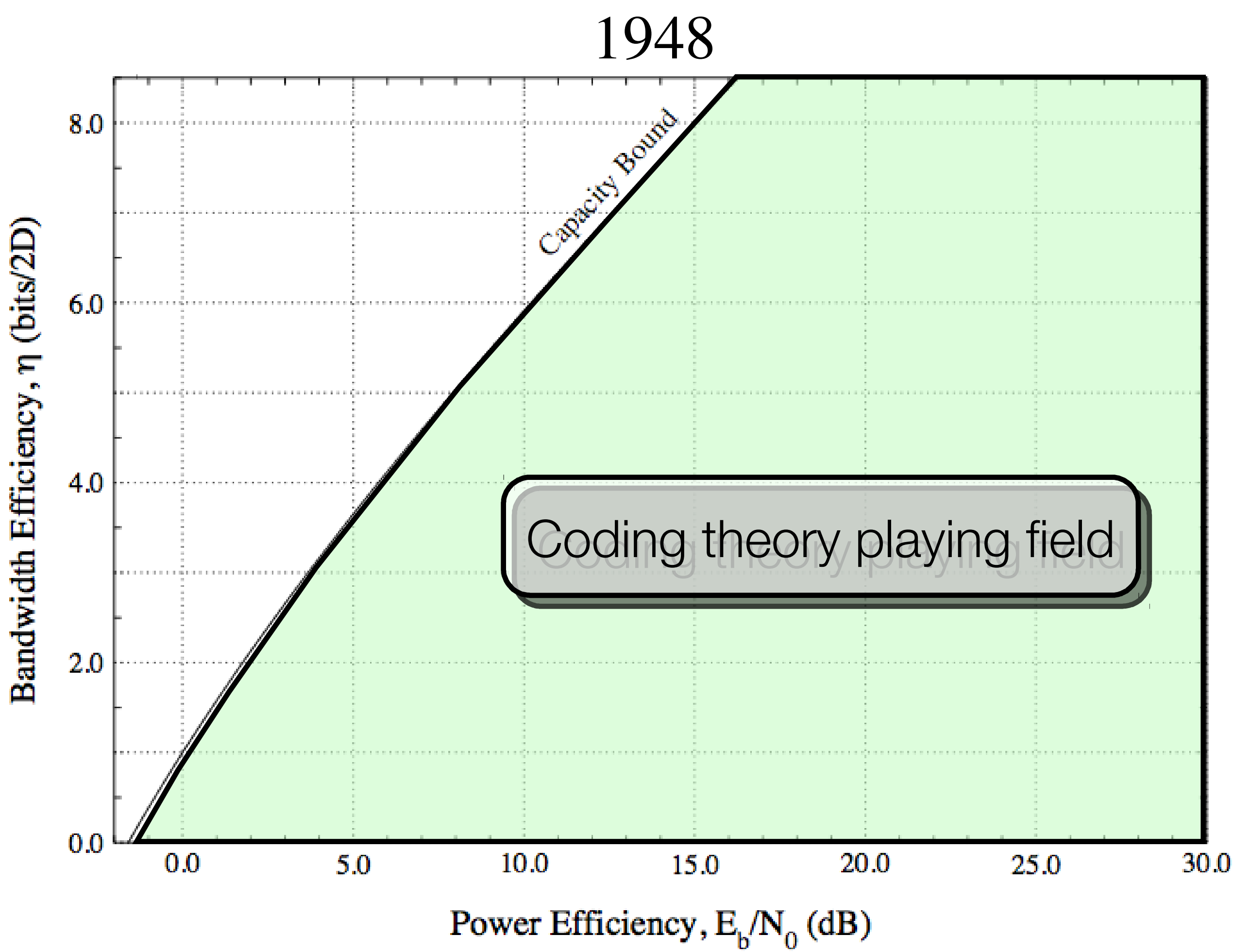
1948



Shannon's capacity theorem lower bounds the channel quality for which there exists a code that can achieve arbitrarily reliable transmission. For the AWGN channel, this lower bound is

$$E_b/N_o > \frac{2^\eta - 1}{\eta}$$

Shannon's Legacy



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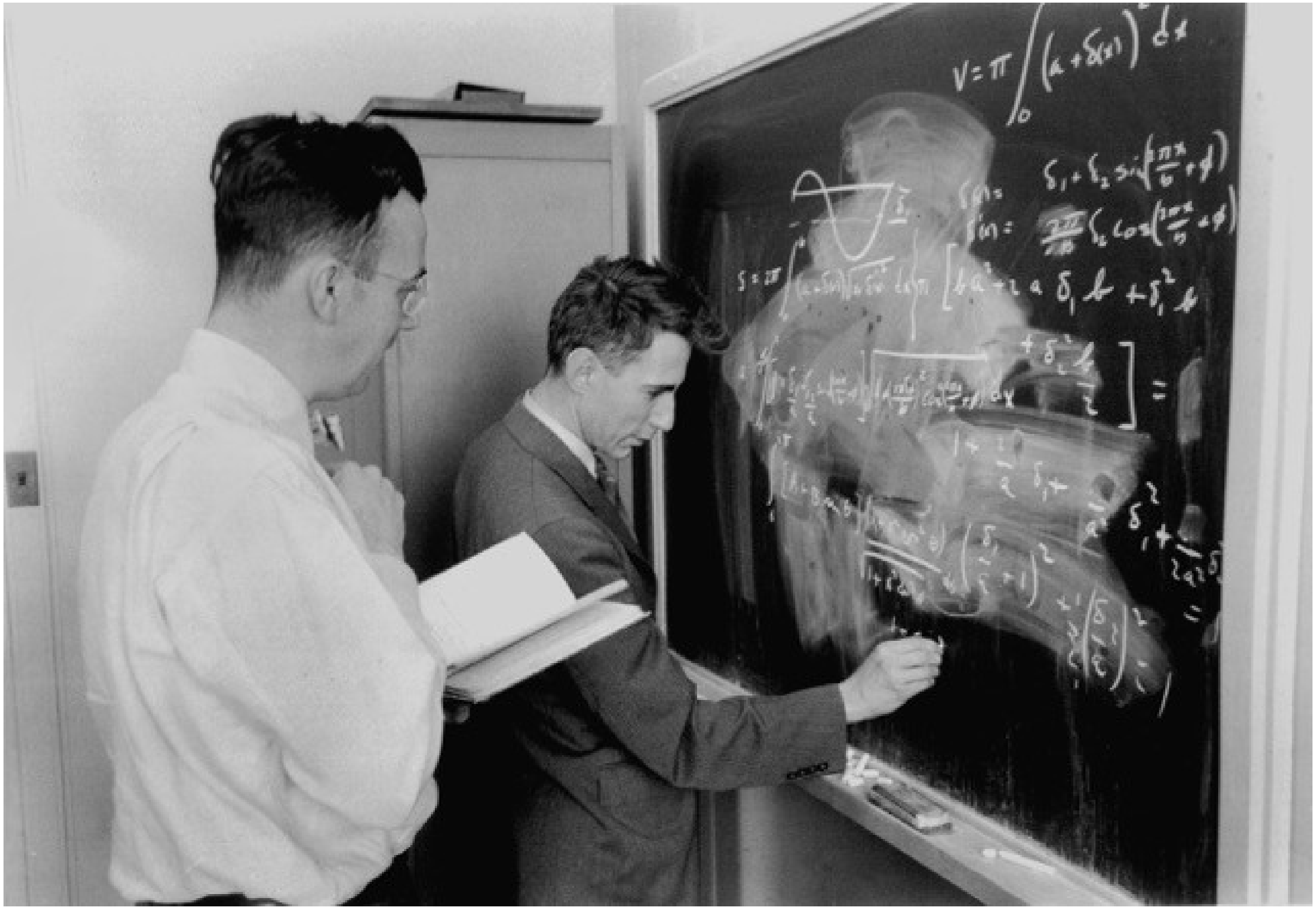
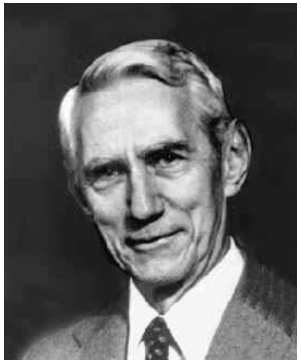


The coding dilemma

- Shannon showed that **random codes** with large block length can **achieve capacity**, but...



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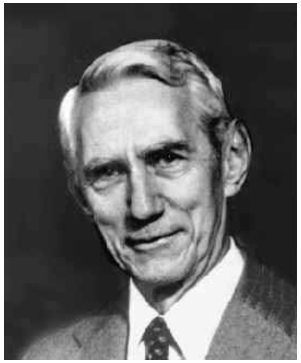

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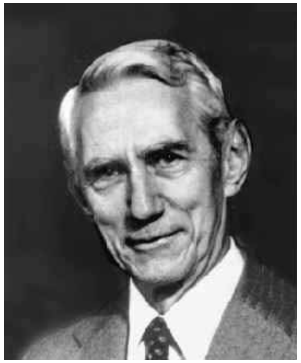

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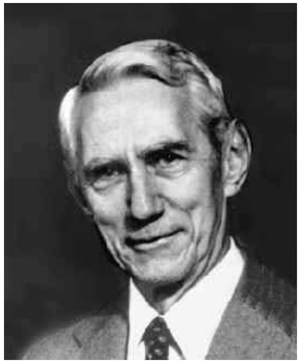

"Almost all codes are good... except those we can think of."

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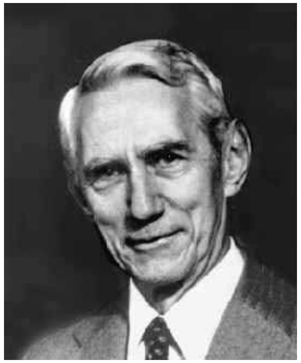
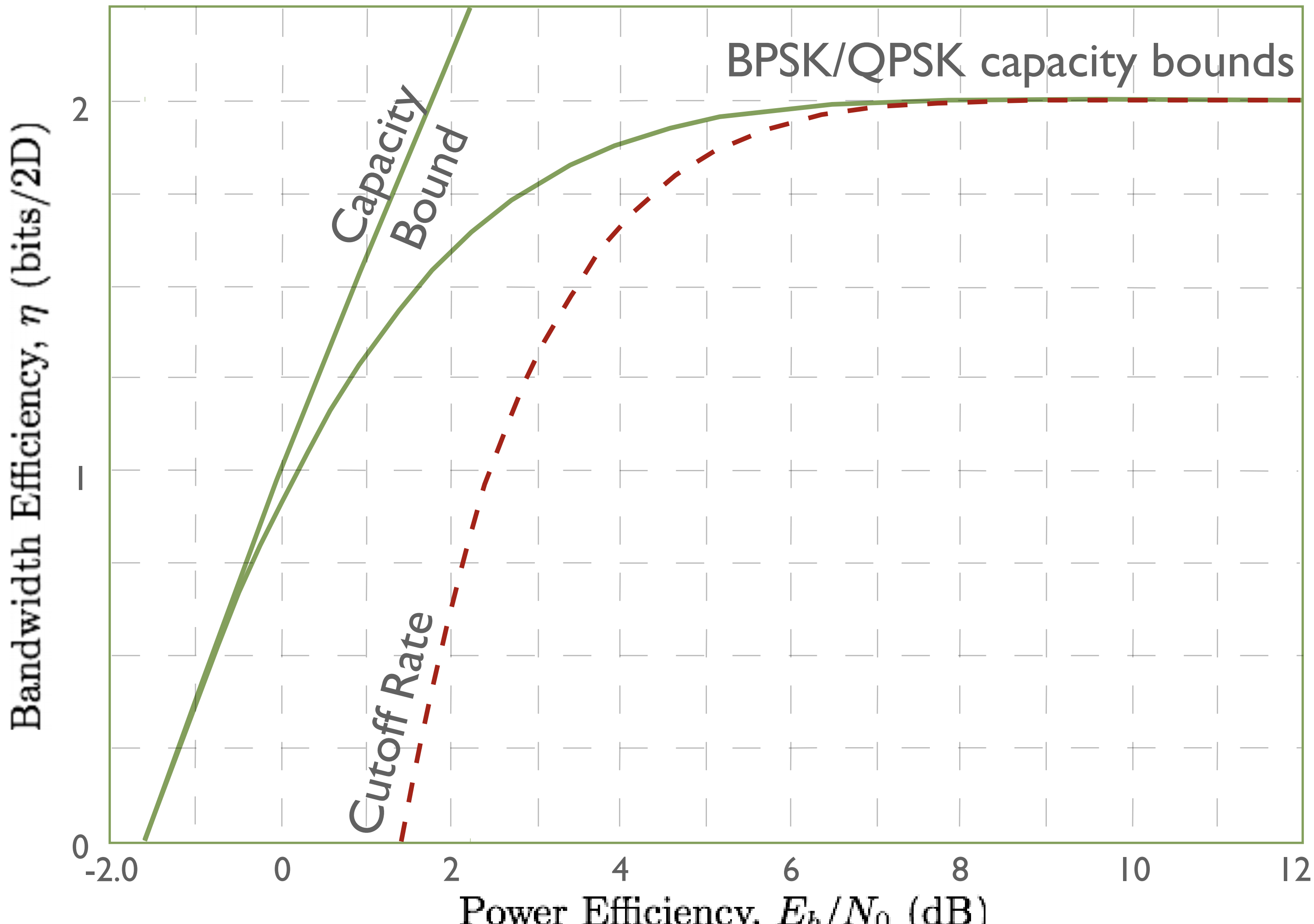


"Almost all codes are good... except those we can think of."

Solution: Construct random-like codes with just enough structure to allow efficient decoding

➡ Modern Coding Theory

LDPC Codes: motivation (for a target BER 10^{-5})



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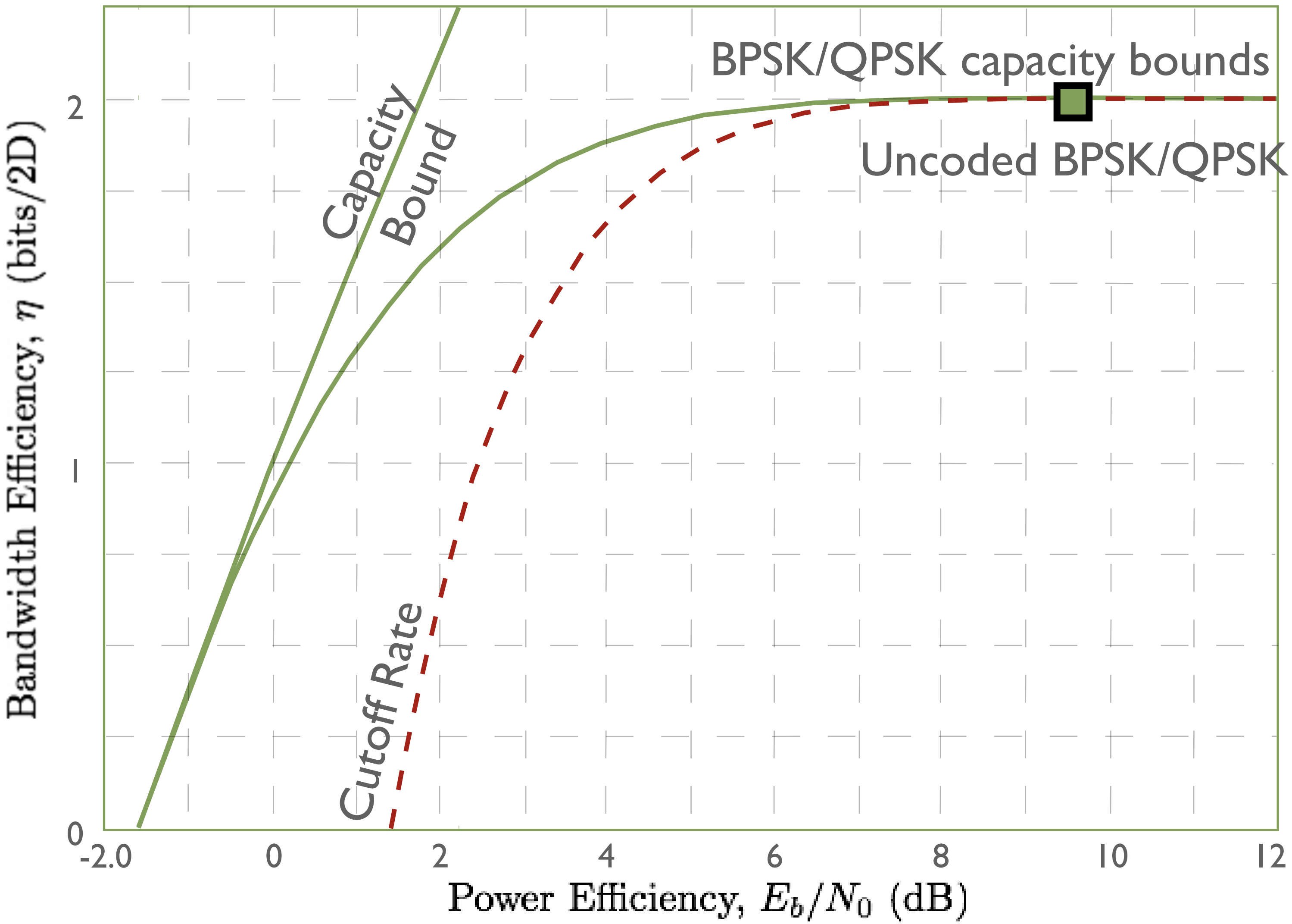

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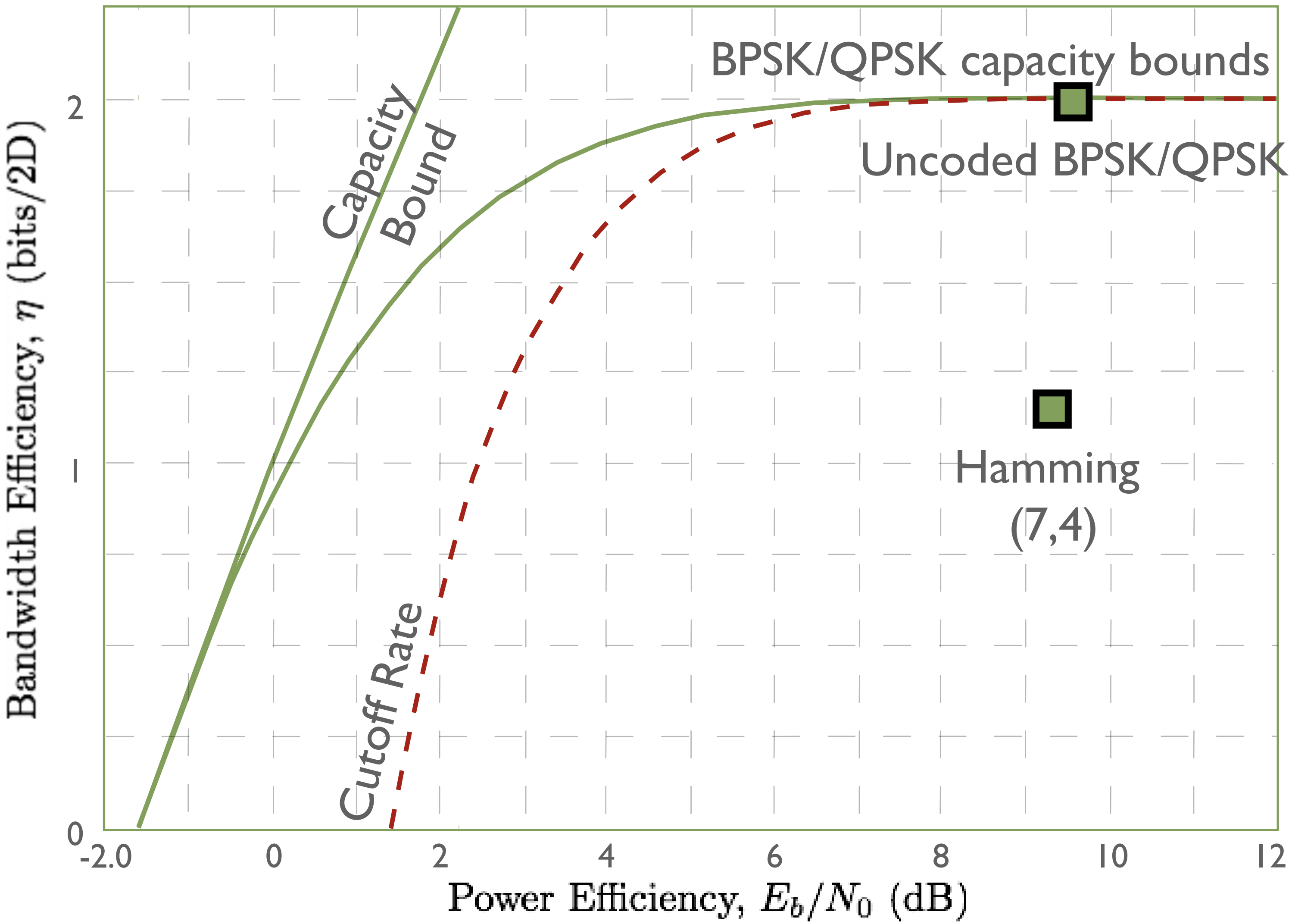

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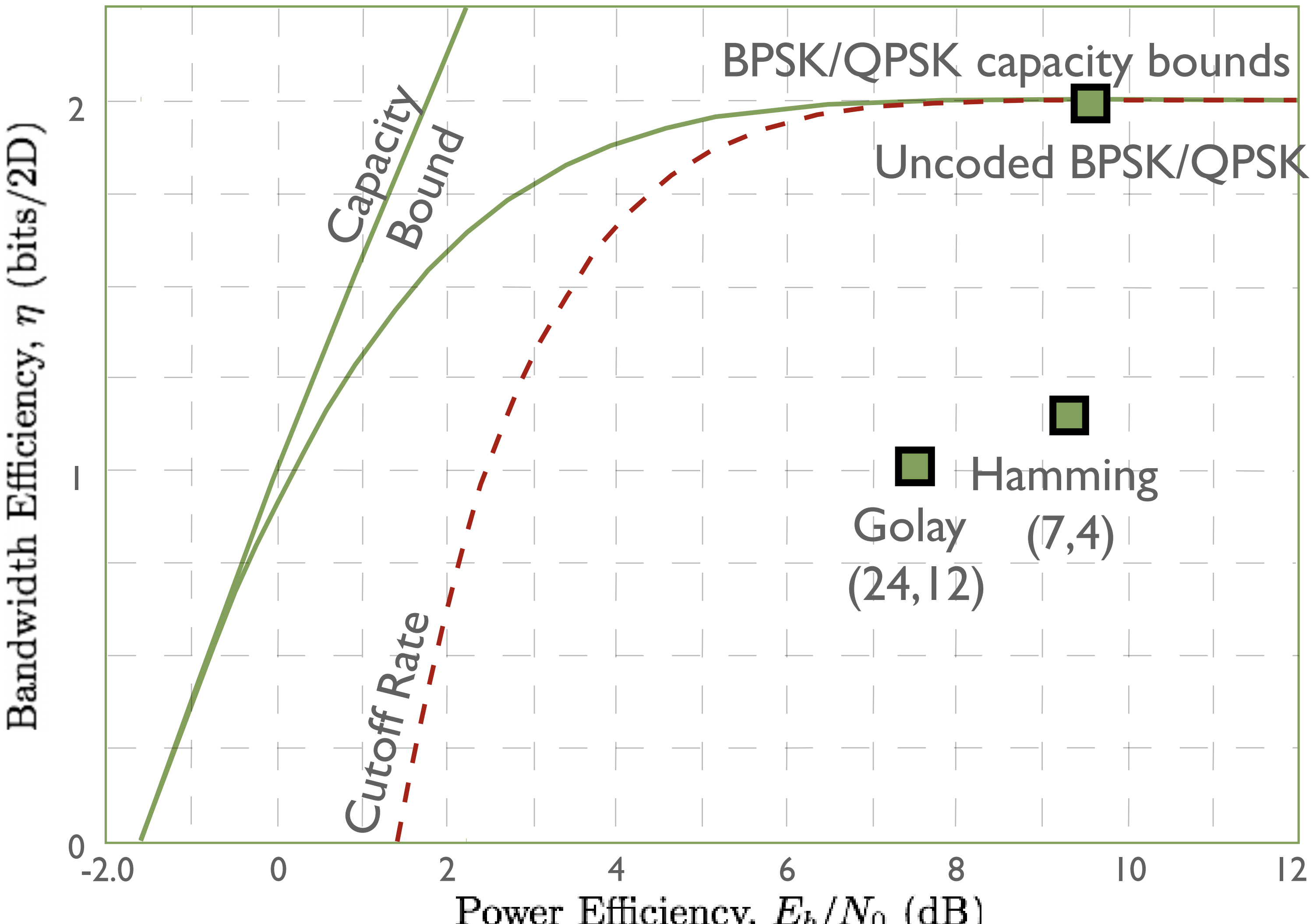

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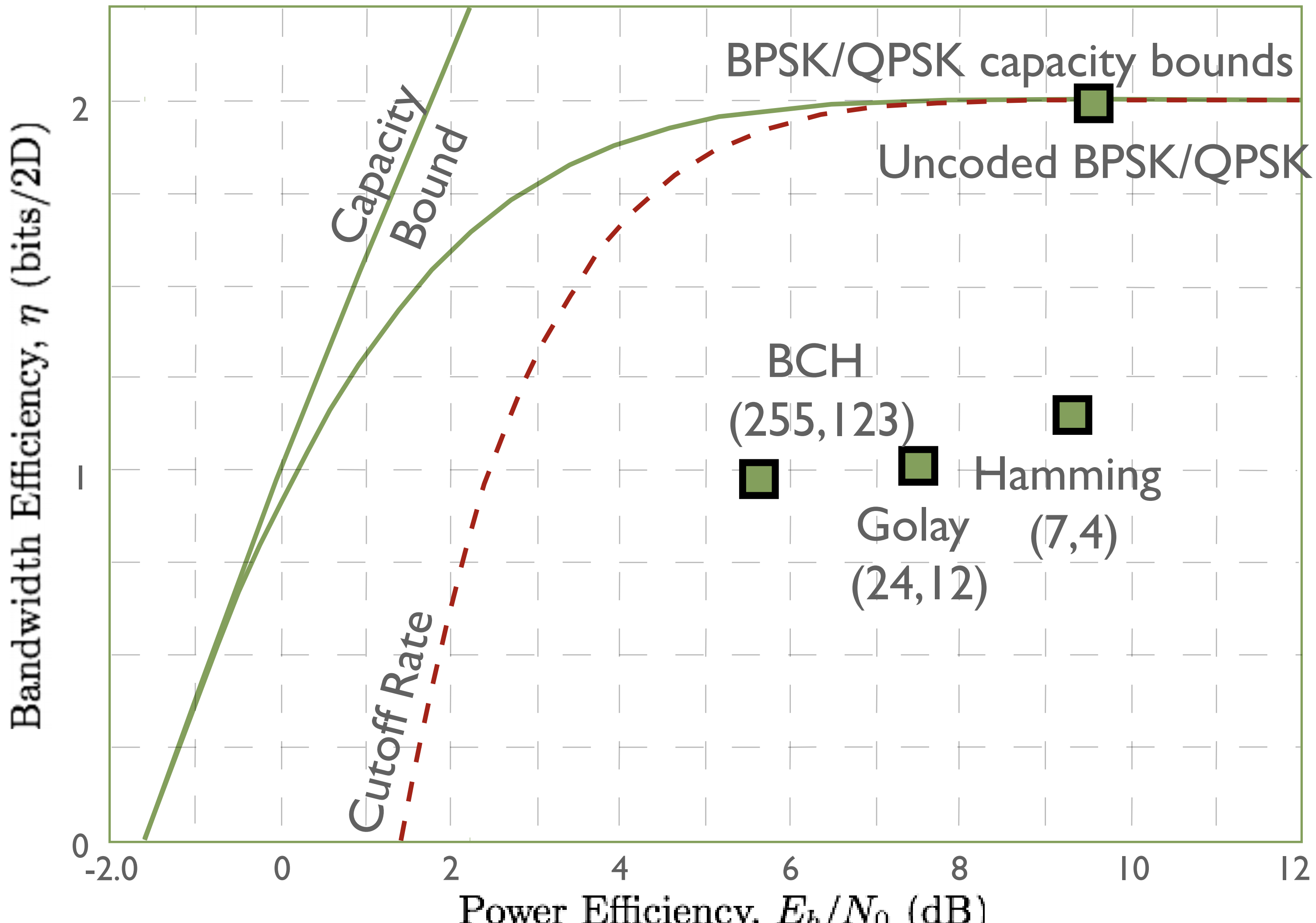

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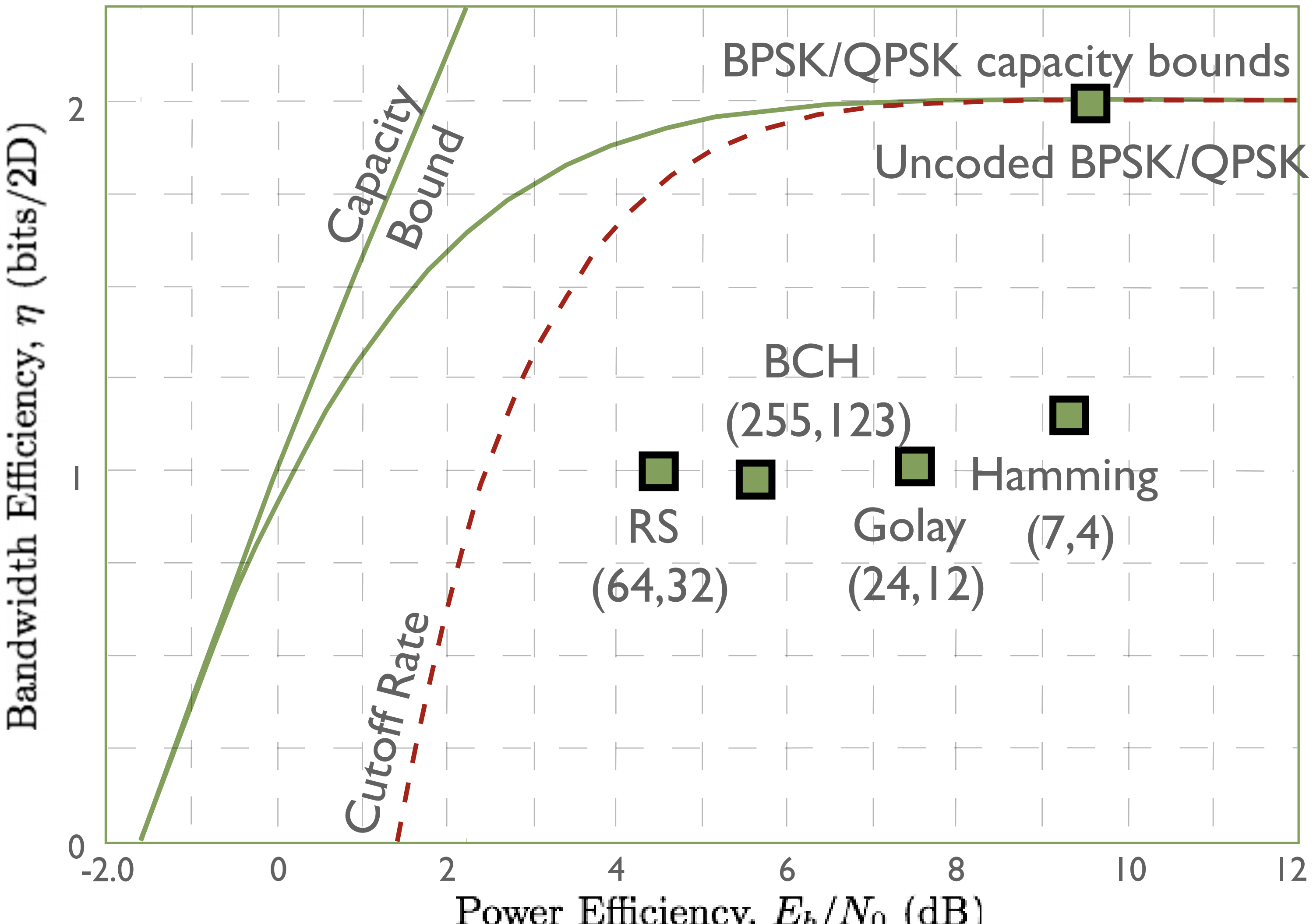

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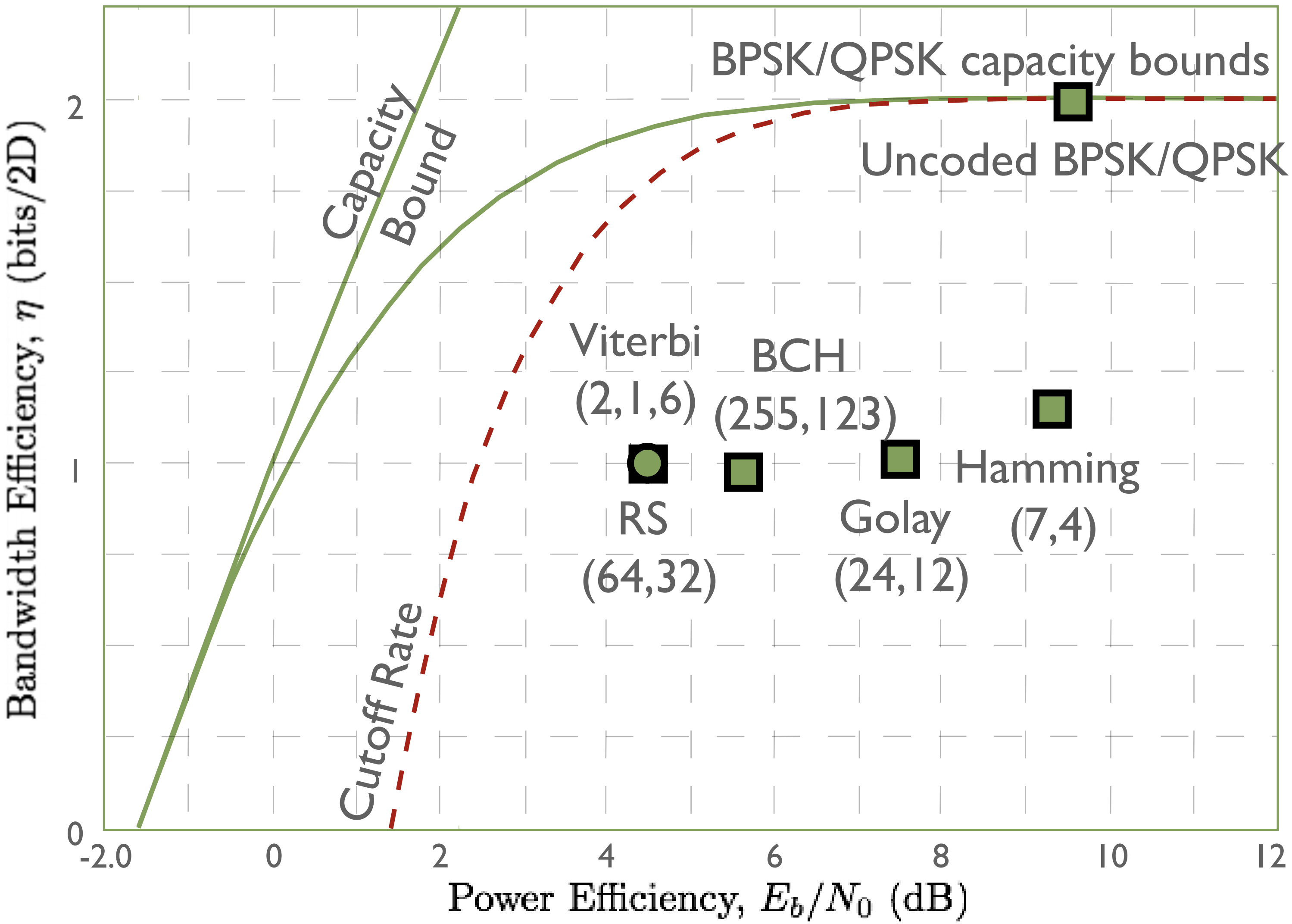

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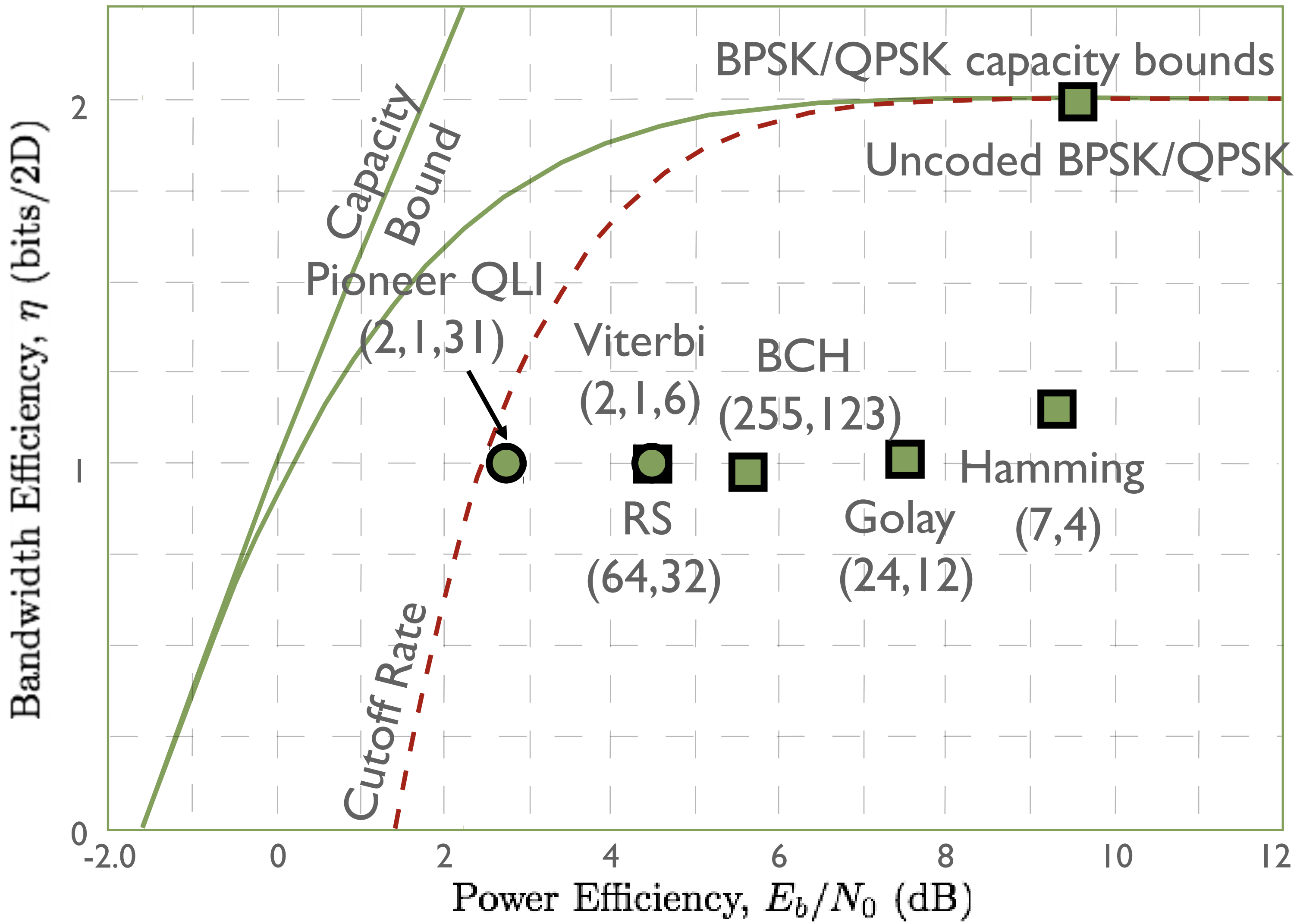

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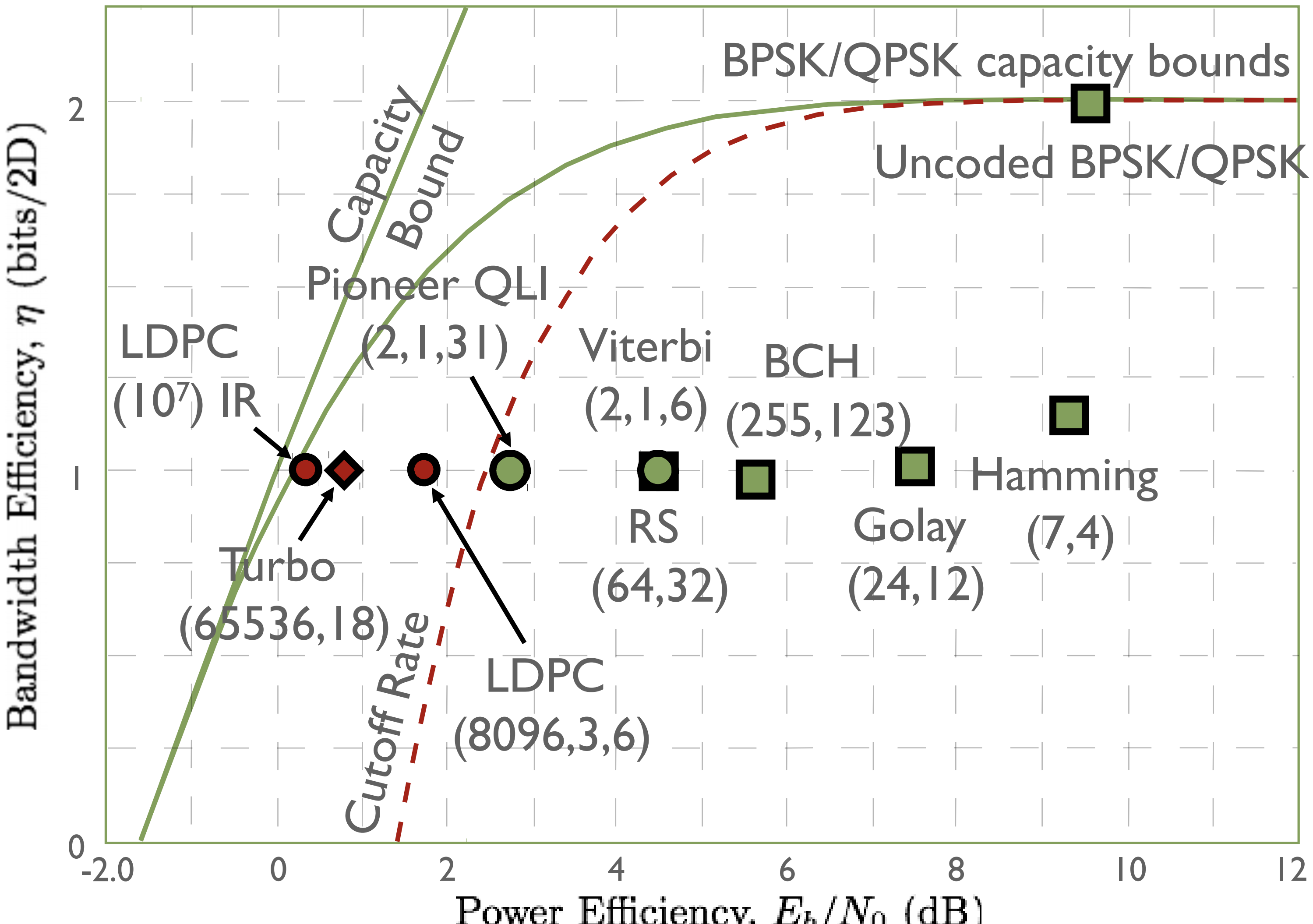
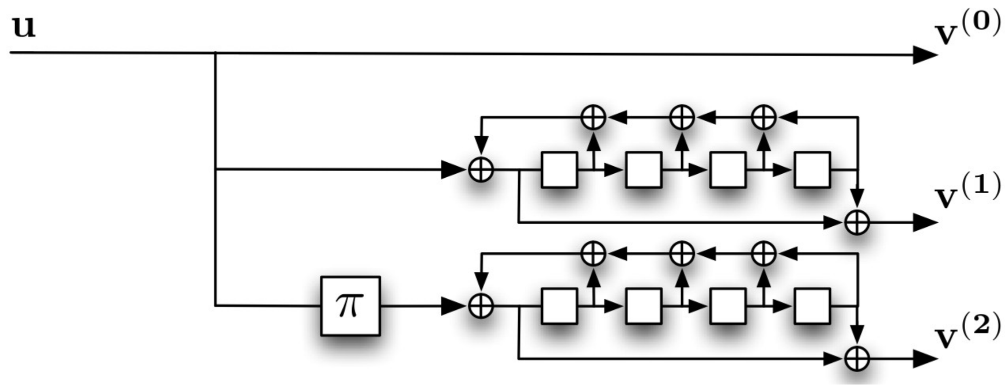


Random-like codes (2000s - today)

- Turbo codes use a long pseudorandom interleaver

➡ 3G and 4G telephony standards HSPA, EV-DO, LTE, satellite DVB-RCS, Mars Reconnaissance Rover, WiMAX, and so on.



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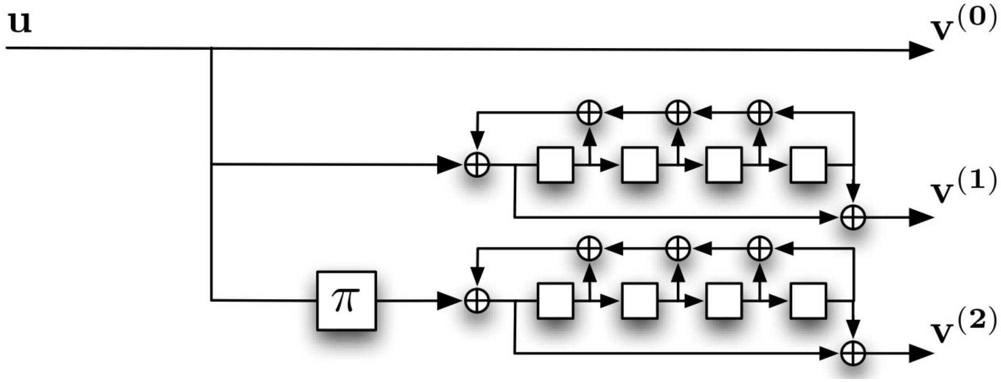
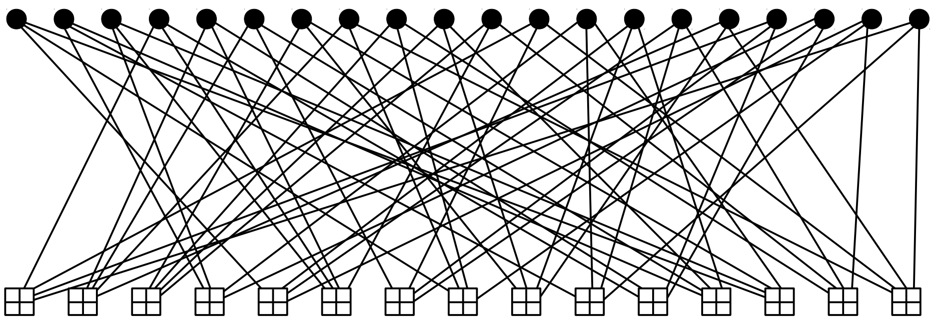


- Low-density parity-check (LDPC) codes are defined on a large sparse graph

➡ DVB-S2, ITU-T G.hn standard (data networking over power lines, phone lines, and coaxial cables), 10GBase-T Ethernet, Wi-Fi standards 802.11, and so on.



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LDPC Block Codes

Definition by parity-check matrix: [Gallager, '62]

Bipartite graph representation: [Tanner, '81]

$$
\mathbf{H} = \left[\begin{array}{cccccccccccccccccccc}
0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&1\\
0&0&0&0&1&1&0&1&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&1&0&1&1&1&0&0&0&0&0&0&0&0&0\\
0&1&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&1&0\\
\hline
0&0&1&1&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&1&1&0&0\\
1&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&1&0&0&1&0&0&0&1&0&1&0&0&0&0\\
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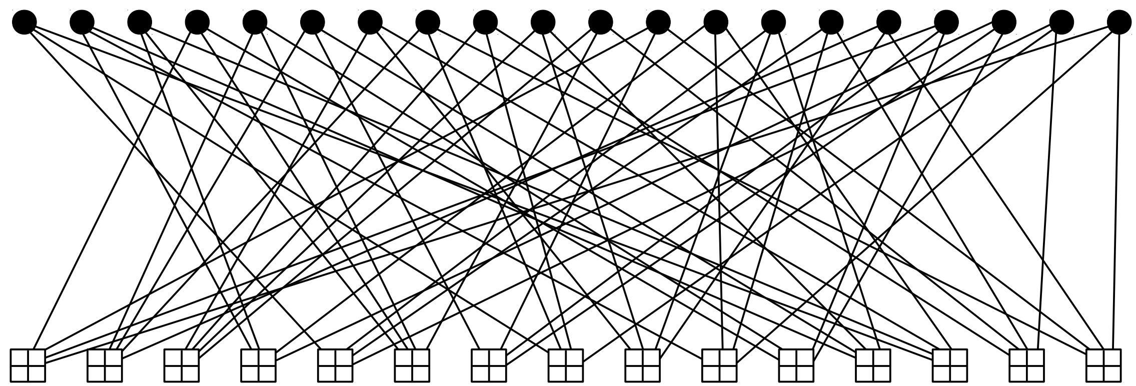
\end{array}\right]_{15 \times 20}
$$

$n = 20$ **variable nodes** of degree $J = 3$



$l = 15$ **check nodes** of degree $K = 4$

Code: $\{\mathbf{v} \mid \mathbf{v}\mathbf{H}^{\mathrm{T}} = \mathbf{0}\}$

(J,K)-regular LDPC block code: $R \geq 1 - \dfrac{J}{K}$

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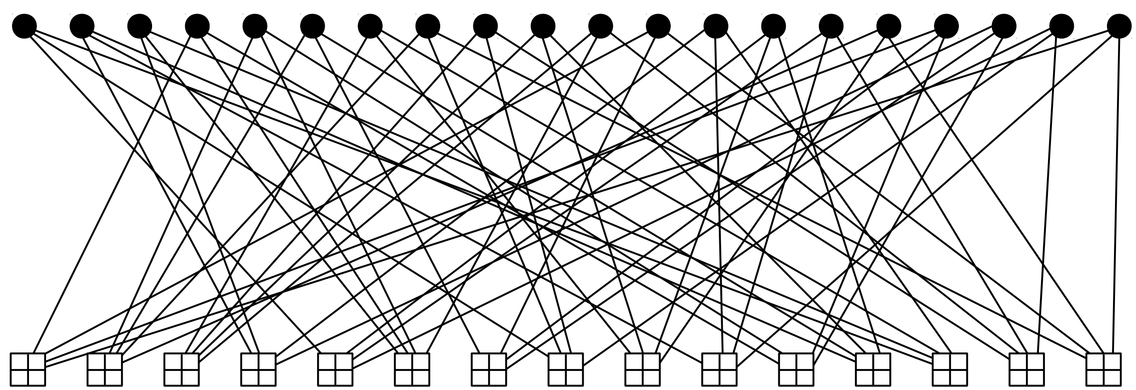

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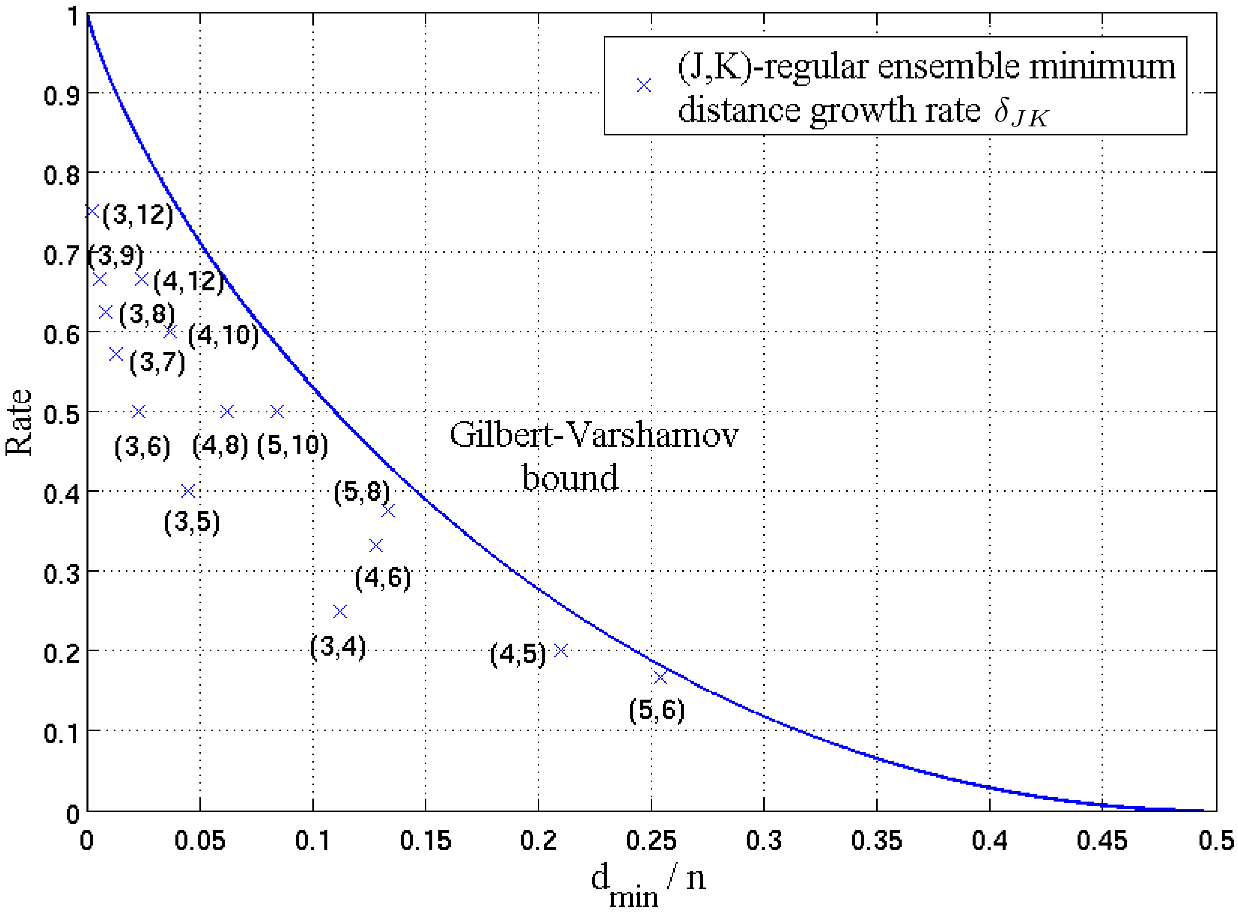
(J,K)-regular LDPC block code: $R \geq 1 - \frac{J}{K}$

- Graph-based codes can be decoded **iteratively** with **low complexity** by exchanging messages in the graph using **Belief Propagation (BP).**

Minimum Distance Growth Rates of (J,K)-Regular LDPC Block Code Ensembles

- For an **asymptotically good** code ensemble, the minimum distance grows linearly with the block length n



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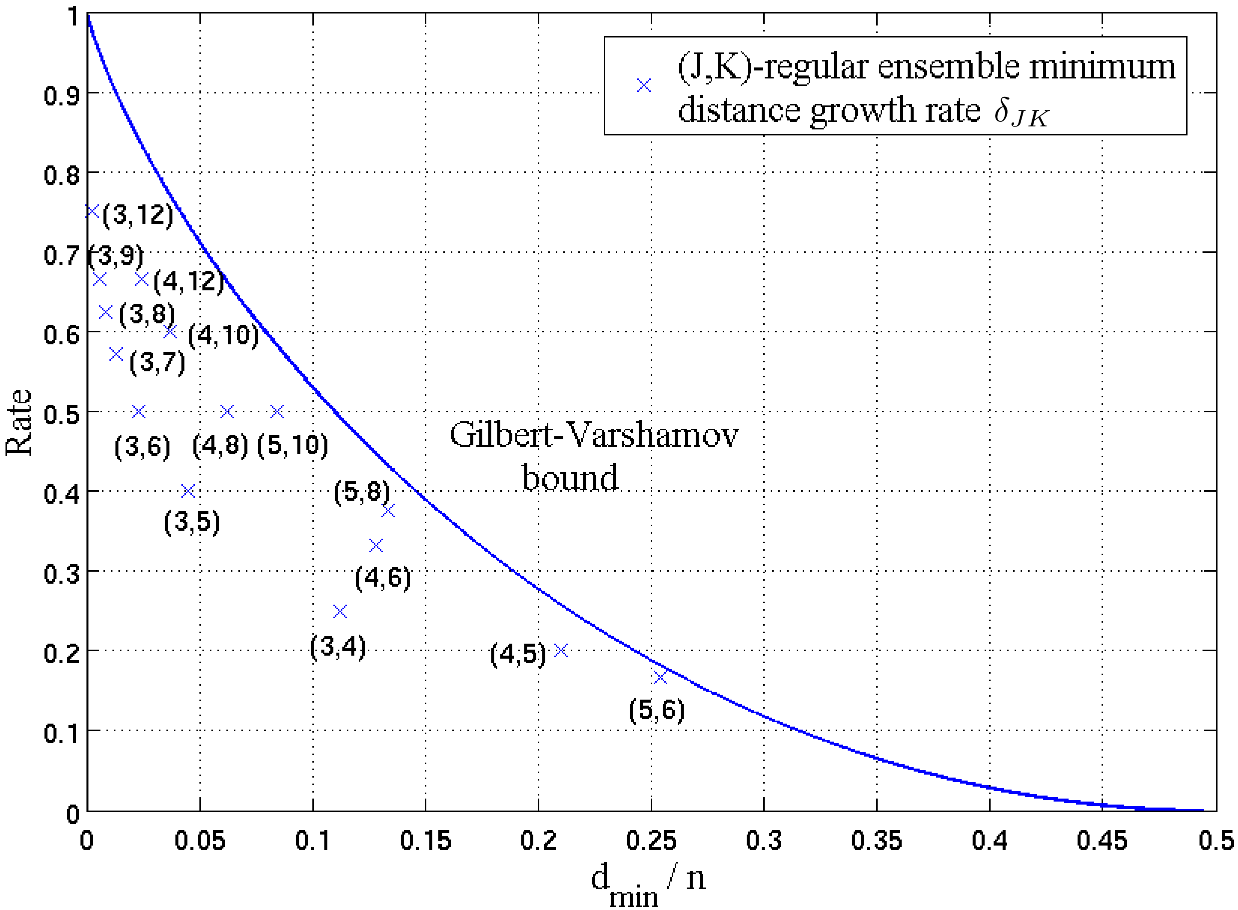

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where δ_{JK} is called the **typical minimum distance ratio**, or **minimum distance growth rate**.

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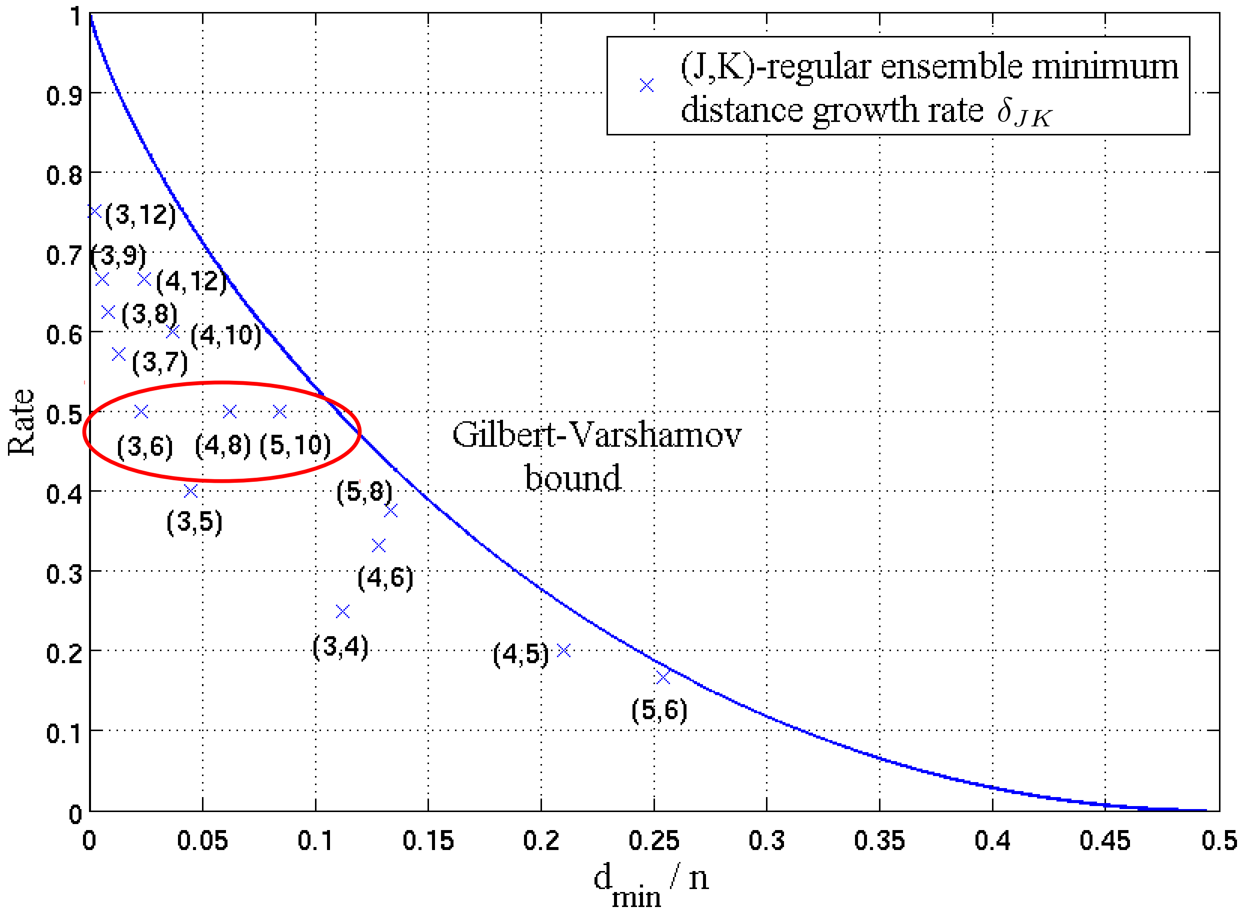

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- As the density of (J,K)-regular ensembles increases, δ_{JK} **approaches the Gilbert-Varshamov bound**.

Thresholds of (J,K)-regular LDPC Block Code Ensembles

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- Iterative decoding thresholds can be calculated for (J,K)-regular LDPC block code ensembles using density evolution (DE).

BEC thresholds

J	K	Rate	ε^*	ε_{Sh}
3	6	0.5	0.429	0.5
4	8	0.5	0.383	0.5
5	10	0.5	0.341	0.5
3	5	0.4	0.517	0.6
4	6	0.333	0.506	0.667
3	4	0.25	0.647	0.75

AWGNC thresholds

J	K	Rate	$(E_b/N_0)^*$	$(E_b/N_0)_{\text{Sh}}$
3	6	0.5	1.11	0.184
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4	6	0.333	1.67	-0.480
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- There exists a relatively **large gap to capacity**.
- Iterative decoding thresholds get **further from capacity** as the graph **density increases**.

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Protographs (Matrix Description)

- Large LDPC codes can be obtained from a small **base parity-check matrix B** by replacing each nonzero entry in **B** with an M x M **permutation matrix**, where M is the **lifting factor**.

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{3\times 6} \Rightarrow \mathbf{H} = \begin{bmatrix} \mathbf{\Pi}_{1,1} & \mathbf{\Pi}_{1,2} & \mathbf{\Pi}_{1,3} & \mathbf{\Pi}_{1,4} & \mathbf{\Pi}_{1,5} & \mathbf{0} \\ \mathbf{\Pi}_{2,1} & \mathbf{0} & \mathbf{\Pi}_{2,3} & \mathbf{\Pi}_{2,4} & \mathbf{\Pi}_{2,5} & \mathbf{\Pi}_{2,6} \\ \mathbf{0} & \mathbf{\Pi}_{3,2} & \mathbf{\Pi}_{3,3} & \mathbf{0} & \mathbf{0} & \mathbf{\Pi}_{3,6} \end{bmatrix}_{3M\times 6M}$$

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Example: Irregular code with M = 4

$$\mathbf{H} = \left[\begin{array}{cccc|cccc|cccc|cccc|cccc|cccc}
0&0&1&0&1&0&0&0&0&0&1&0&1&0&0&0&0&0&1&0&0&0&0&0\\
1&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0\\
0&0&0&1&0&0&1&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0\\
0&1&0&0&0&1&0&0&1&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0\\
\hline
1&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&1&0&0&0&0&1&0\\
0&1&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&1\\
0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&1&0&0&0&1&0&0&0\\
0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&1&0&0&1&0&0&1&0&0\\
\hline
0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&1&0
\end{array}\right]$$

- length $6M$ = 24
- rate R = 1/2

Protographs (Matrix Description)

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- Large LDPC codes can be obtained from a small **base parity-check matrix B** by replacing each nonzero entry in **B** with an M x M **permutation matrix**, where M is the **lifting factor**.

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{3\times 6} \Rightarrow \mathbf{H} = \begin{bmatrix} \mathbf{\Pi}_{1,1} & \mathbf{\Pi}_{1,2} & \mathbf{\Pi}_{1,3} & \mathbf{\Pi}_{1,4} & \mathbf{\Pi}_{1,5} & \mathbf{0} \\ \mathbf{\Pi}_{2,1} & \mathbf{0} & \mathbf{\Pi}_{2,3} & \mathbf{\Pi}_{2,4} & \mathbf{\Pi}_{2,5} & \mathbf{\Pi}_{2,6} \\ \mathbf{0} & \mathbf{\Pi}_{3,2} & \mathbf{\Pi}_{3,3} & \mathbf{0} & \mathbf{0} & \mathbf{\Pi}_{3,6} \end{bmatrix}_{3M\times 6M}$$

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1&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0 \\
0&0&0&1&0&0&1&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0 \\
0&1&0&0&0&1&0&0&1&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0 \\ \hline
1&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&1&0&0&0&0&1&0 \\
0&1&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&1 \\
0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&1&0&0&0&1&0&0&0 \\
0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&1&0&0&1&0&0&1&0&0 \\ \hline
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0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&1 \\
0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\
0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&1&0
\end{array}\right]$$

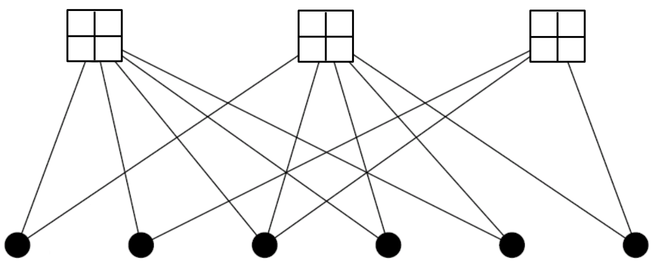
- length $6M$ = 24
  rate R = 1/2
- Irregular codes have variable row and column weights (check node and variable node degrees)

Protographs (Graphical Description)

- Protographs are often represented using a bipartite **Tanner graph**

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{3\times 6} \Longleftrightarrow$$

3 check nodes



6 variable nodes

[Tho05] J. Thorpe, "Low-Density Parity-Check (LDPC) codes constructed from protographs", *Jet Propulsion Laboratory INP Progress Report*, Vol. 42-154 Aug. 2003.

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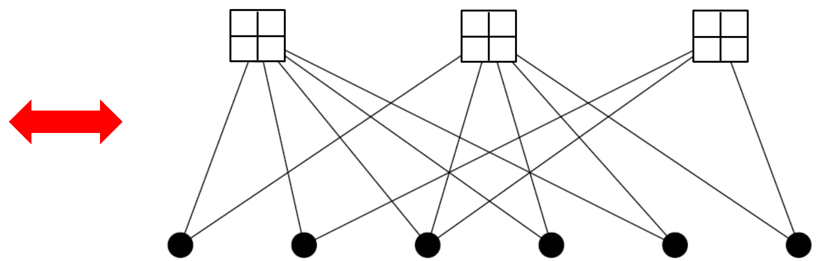

6 variable nodes

- The collection of all possible parity-check matrices with lifting factor M forms a **code ensemble**, where all the codes share a **common structure**

$$\mathbf{H} = \begin{bmatrix} \mathbf{\Pi}_{1,1} & \mathbf{\Pi}_{1,2} & \mathbf{\Pi}_{1,3} & \mathbf{\Pi}_{1,4} & \mathbf{\Pi}_{1,5} & \mathbf{0} \\ \mathbf{\Pi}_{2,1} & \mathbf{0} & \mathbf{\Pi}_{2,3} & \mathbf{\Pi}_{2,4} & \mathbf{\Pi}_{2,5} & \mathbf{\Pi}_{2,6} \\ \mathbf{0} & \mathbf{\Pi}_{3,2} & \mathbf{\Pi}_{3,3} & \mathbf{0} & \mathbf{0} & \mathbf{\Pi}_{3,6} \end{bmatrix}$$

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Quasi-Cyclic LDPC Codes

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Example: protograph construction of a $(2,3)$**-regular** QC-LDPC block code

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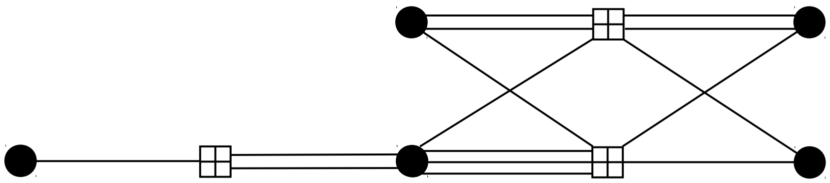
0&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0\\
0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&1&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0\\
\hline
0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0\\
0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0\\
0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&1\\
0&0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0\\
0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&1&0&0&0&0\\
1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0
\end{array}\right]_{14\times 21}$$

$$M = 7$$
$$n = 3M = 21$$
$$R = 1/3$$

For QC codes, the permutation matrices are **shifted identities**

Multi-Edge Protographs

- Protographs can have repeated edges (corresponding to integer values greater than one in **B**)



$$\mathbf{B} = \begin{bmatrix} 2 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & 3 & 1 \end{bmatrix}_{3 \times 5}$$

- Note that this makes no sense without lifting

[DDJA09] D. Divsalar, S. Dolinar, C. Jones, and K. Andrews, "Capacity-approaching protograph codes", *IEEE Journal on Select Areas in Communications*, vol. 27, no. 6 Aug. 2009.

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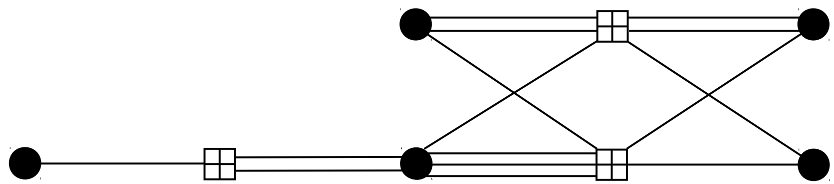

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- Repeated edges in a protograph correspond to using sums of permutation matrices to form LDPC code ensembles

$$\mathbf{H} = \left[\begin{array}{ccc|ccc|ccc|ccc|ccc} 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \end{array}\right]$$

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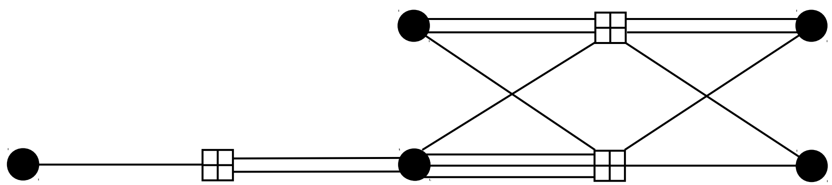

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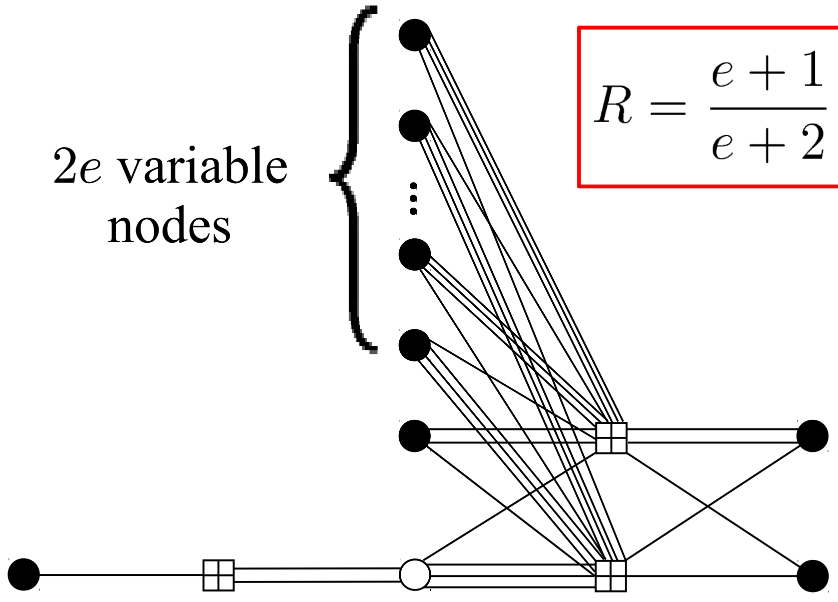
➡ denser graphs!

➡ can also be QC (using **circulant matrices**)!

[DDJA09] D. Divsalar, S. Dolinar, C. Jones, and K. Andrews, "Capacity-approaching protograph codes", *IEEE Journal on Select Areas in Communications*, vol. 27, no. 6 Aug. 2009.

'Good' Protographs

- Ensemble average properties can be easily calculated from a protograph, thus simplifying the construction of 'good' code ensembles.
  - ➡ Iterative decoding thresholds close to capacity for irregular protographs
  - ➡ Minimum distance growing linearly with block length (**asymptotically good**) for regular and some irregular protographs



$$R = \frac{e+1}{e+2}$$

Rate	Threshold $(E_b/N_0)^*$	Capacity $(E_b/N_0)_{\rm Sh}$	Distance growth rate
1/2	0.628	0.187	0.01450
2/3	1.450	1.059	0.00582
3/4	2.005	1.628	0.00323
4/5	2.413	2.040	0.00207
5/6	2.733	2.362	0.00145
6/7	2.993	2.625	0.00108

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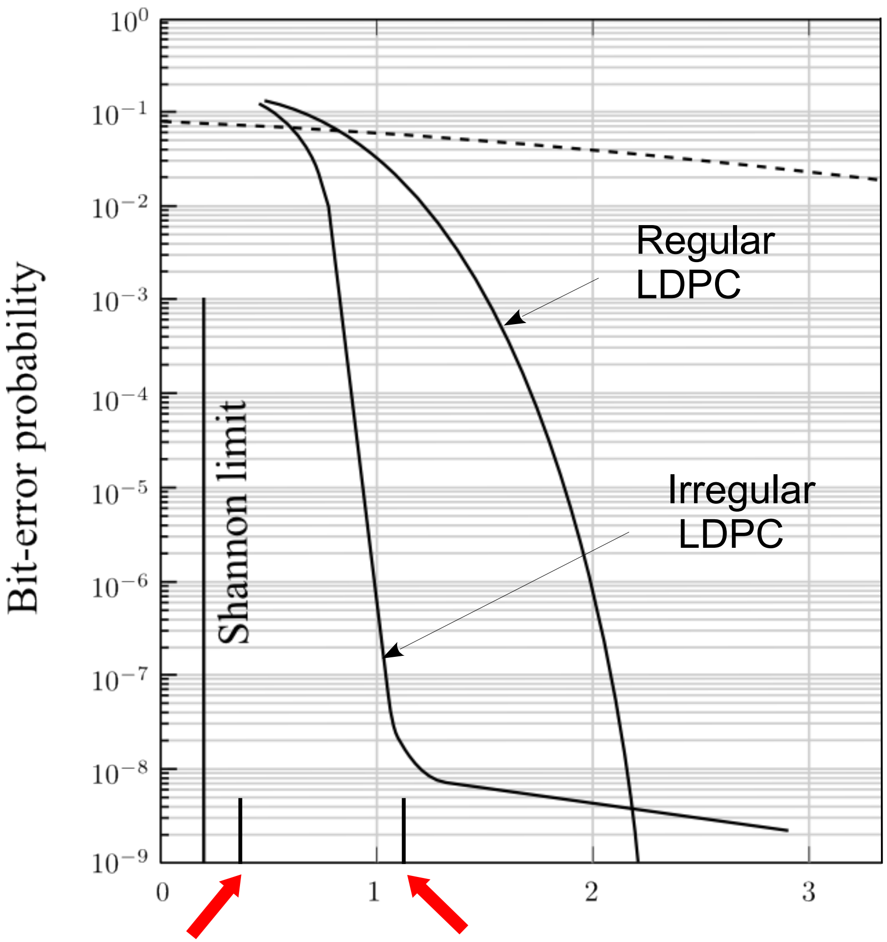
Regular vs. Irregular LDPC codes



irregular threshold

regular threshold

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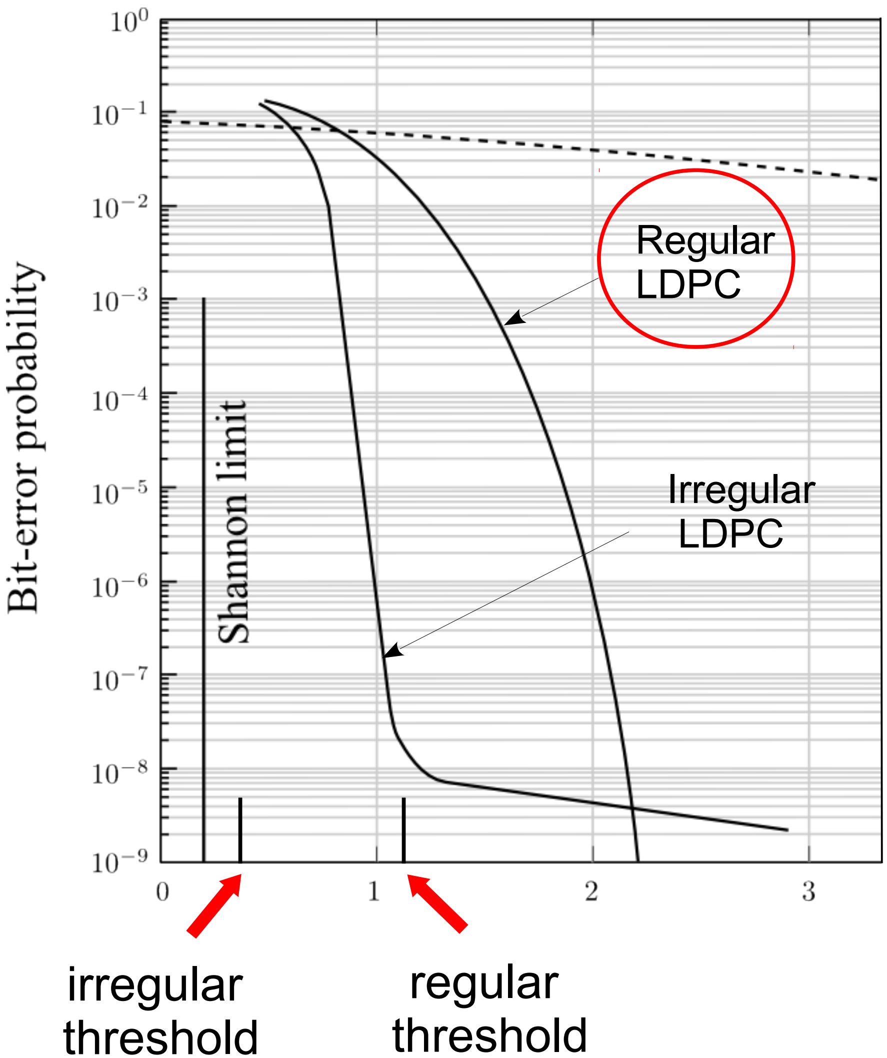

- "Regular" LDPC codes:
  - ✓ structure **aids implementation**
  - ✓ low **error floors**
  - ✗ thresholds **far from capacity**

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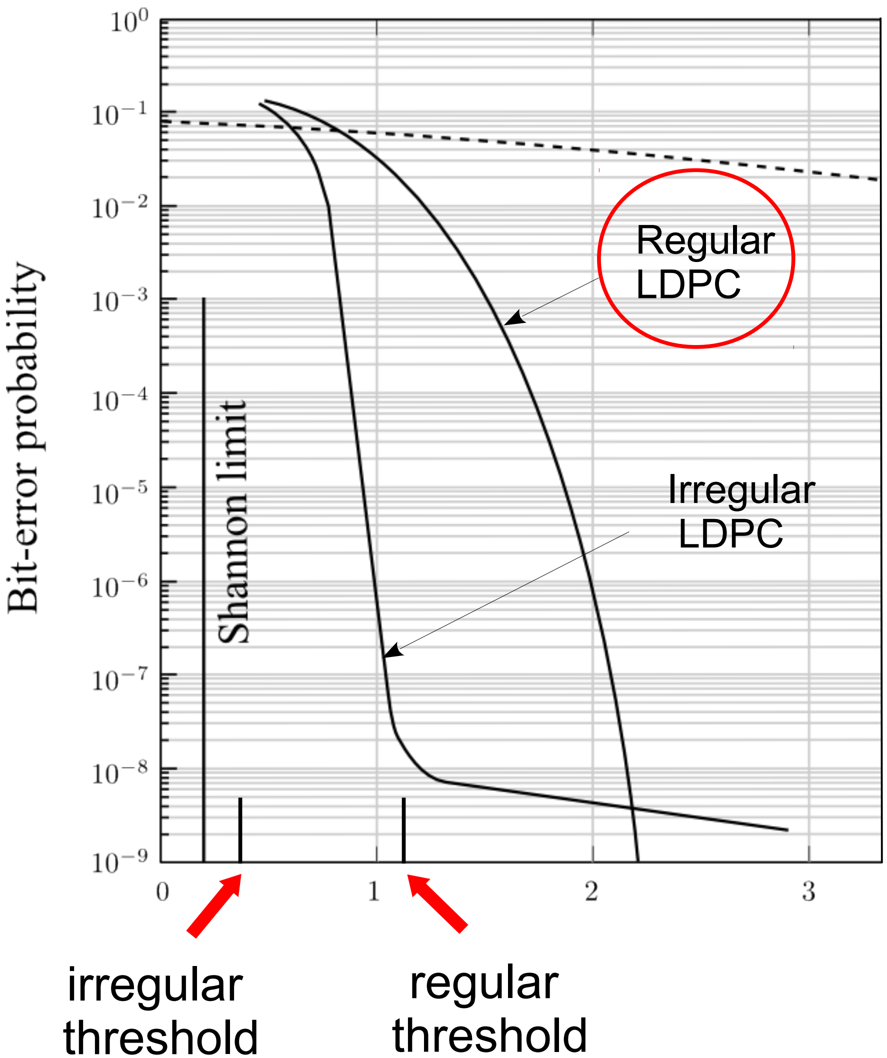

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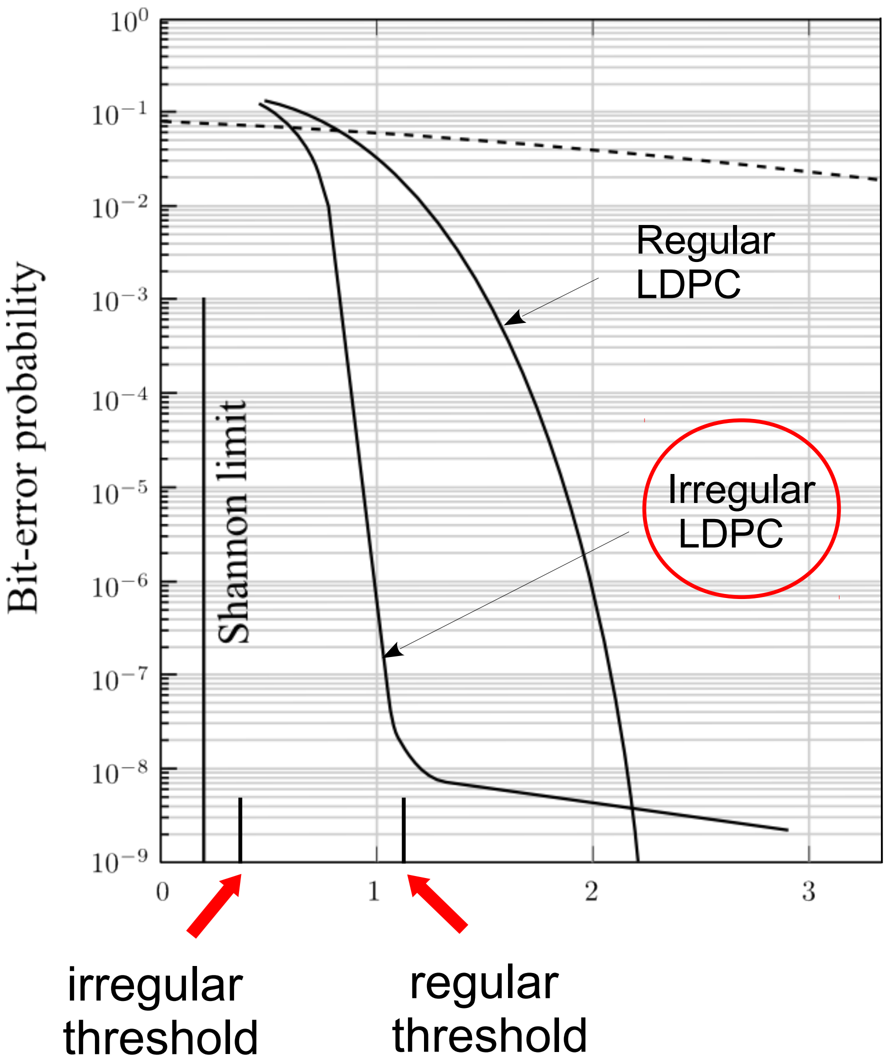

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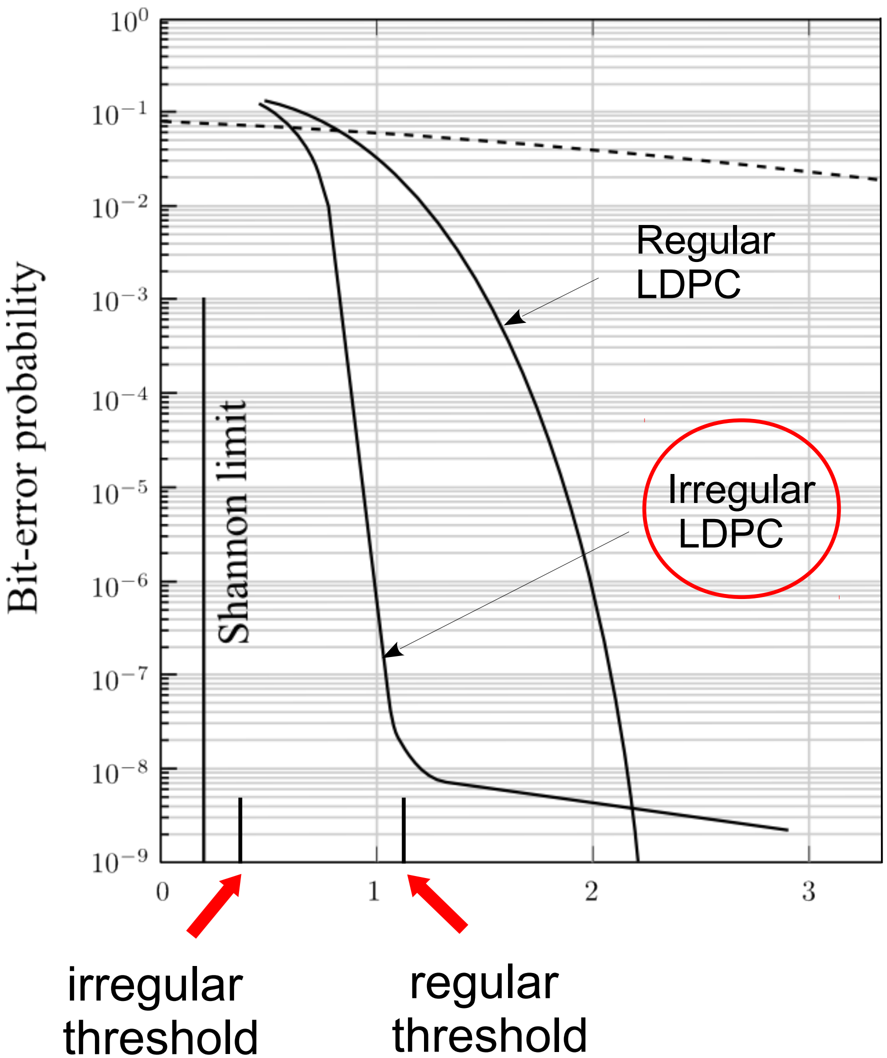

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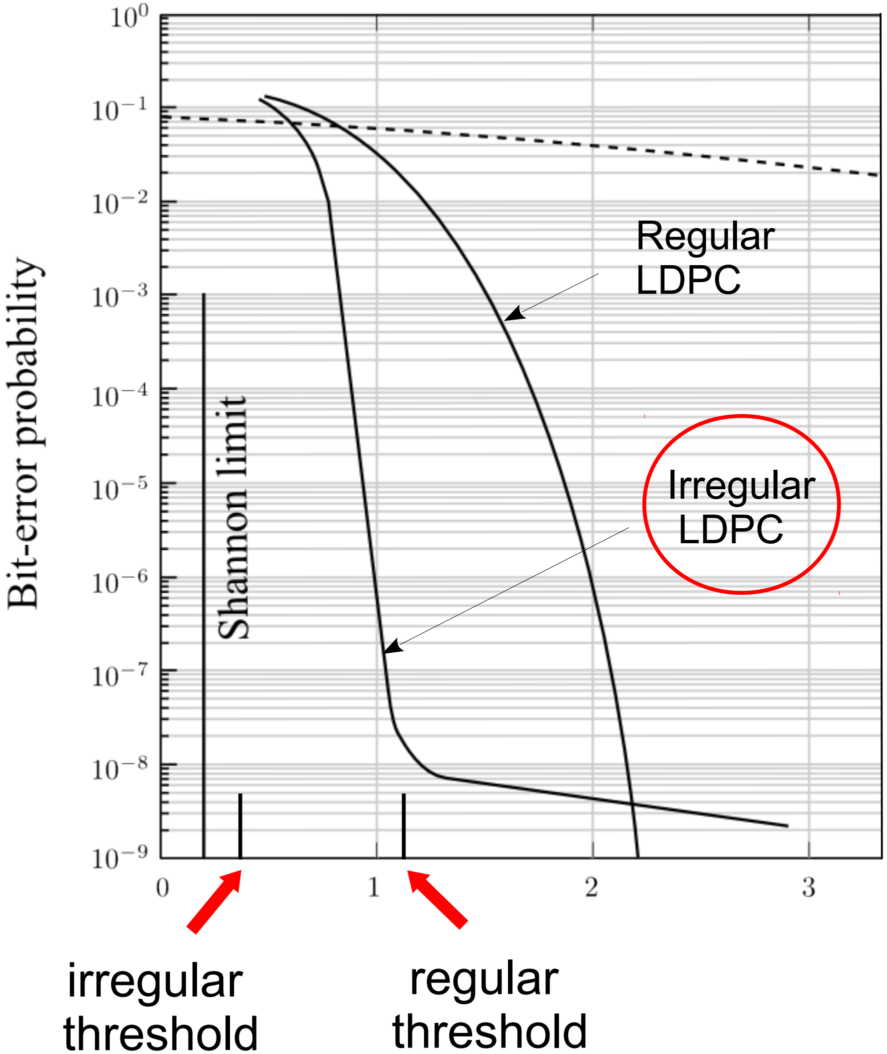

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- **Spatially coupled** LDPC codes combine all of the positive features!

Outline

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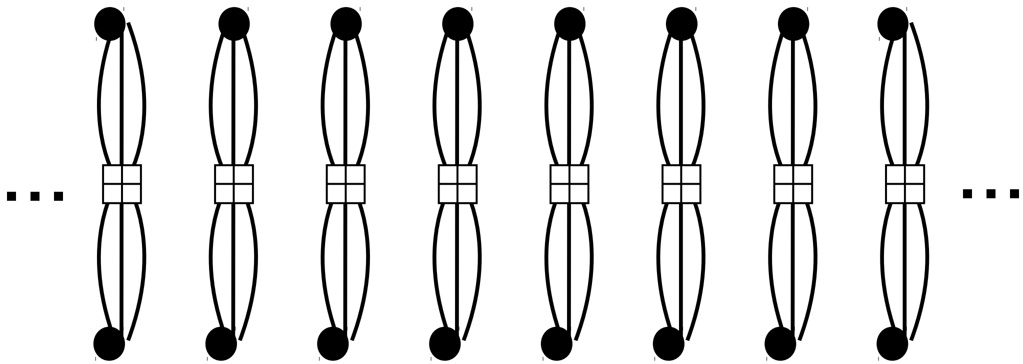
- **LDPC Block Codes**
  - Parity-check matrix and Tanner graph representations, minimum distance bounds, iterative decoding thresholds, protograph-based constructions
- **Spatially Coupled LDPC Codes**
  - Protograph representation, edge-spreading construction, termination
  - Iterative decoding thresholds, threshold saturation, minimum distance
- **Practical Considerations**
  - Window decoding; performance, latency, and complexity comparisons to LDPC block codes; rate-compatibility; implementation aspects

Spatially Coupled Protographs

- Consider transmission of consecutive blocks (protograph representation):



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$(3{,}6)$-regular LDPC-BC base matrix

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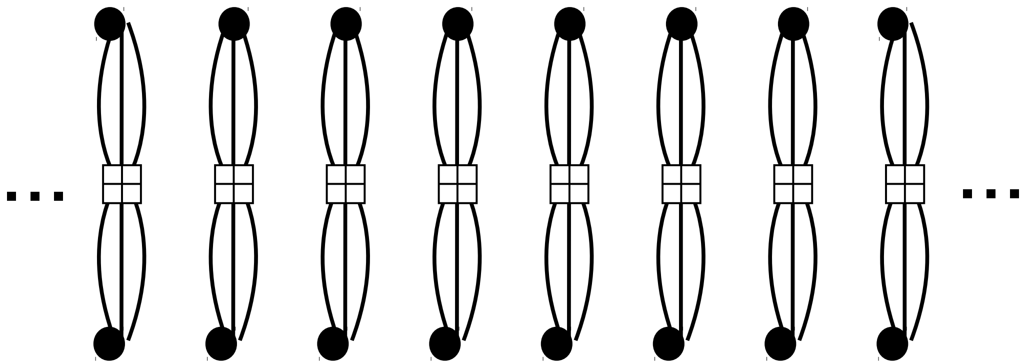

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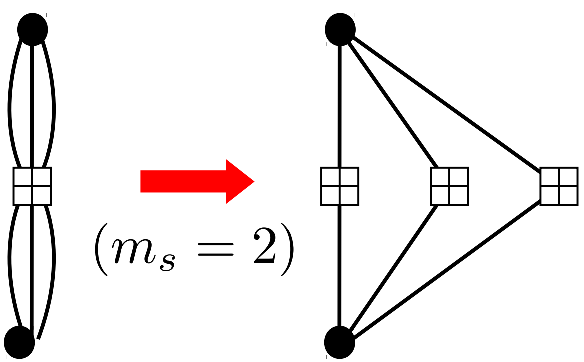

$$\mathbf{B} = \begin{bmatrix} 3 & 3 \end{bmatrix} \xrightarrow{(m_s = 2)} \begin{bmatrix} \mathbf{B}_0 \\ \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

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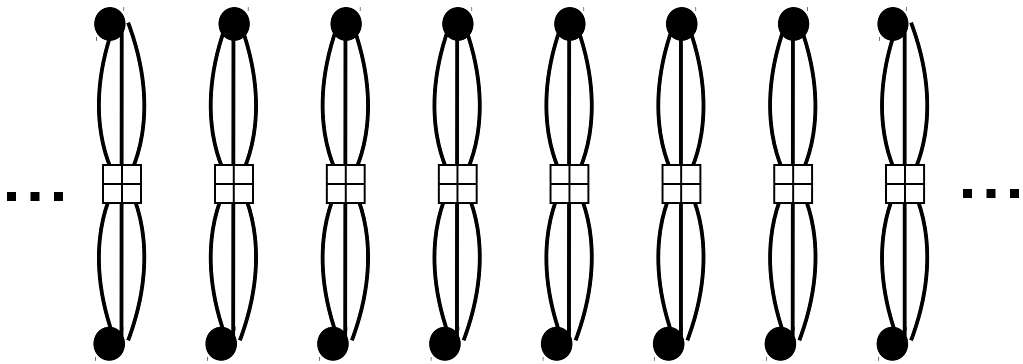

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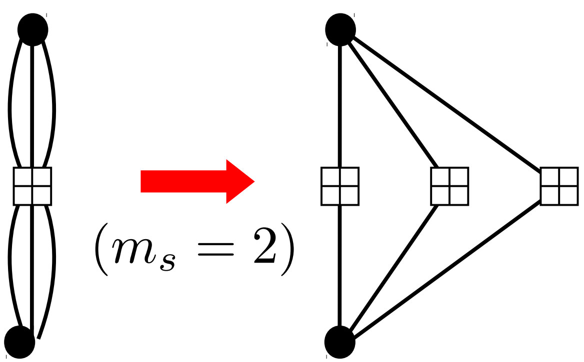

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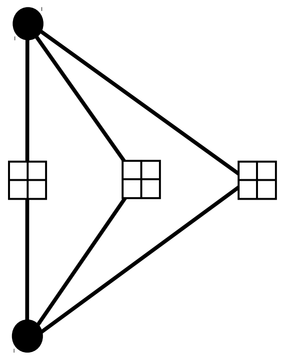
- **Spreading constraint:** $\sum_{i=0}^{m_s} \mathbf{B}_i = \mathbf{B}$ ($\mathbf{B}_i$ has size $b_c \times b_v$)

Spatially Coupled Protographs

- Transmission of consecutive spatially coupled (SC) blocks results in a **convolutional protograph**:



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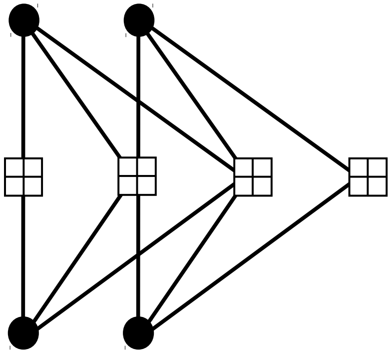

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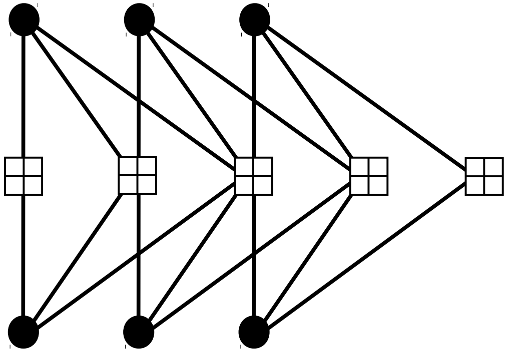

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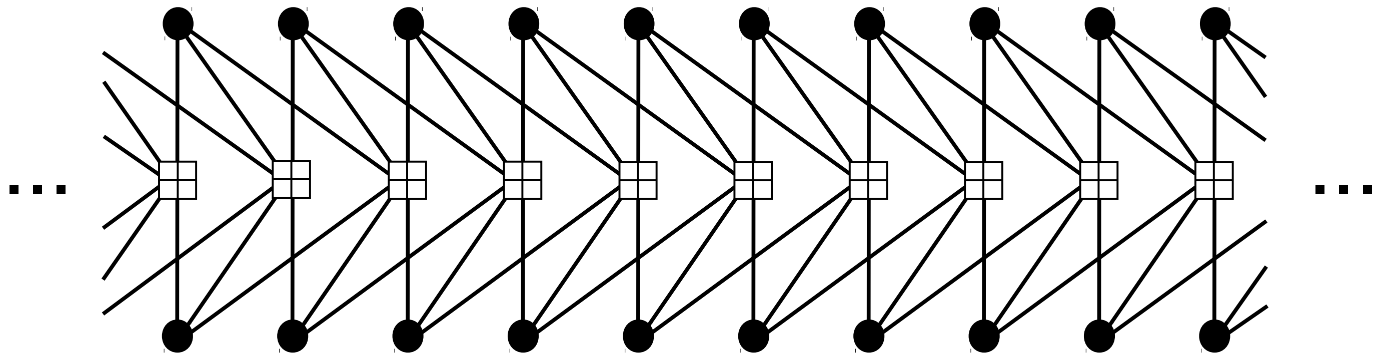

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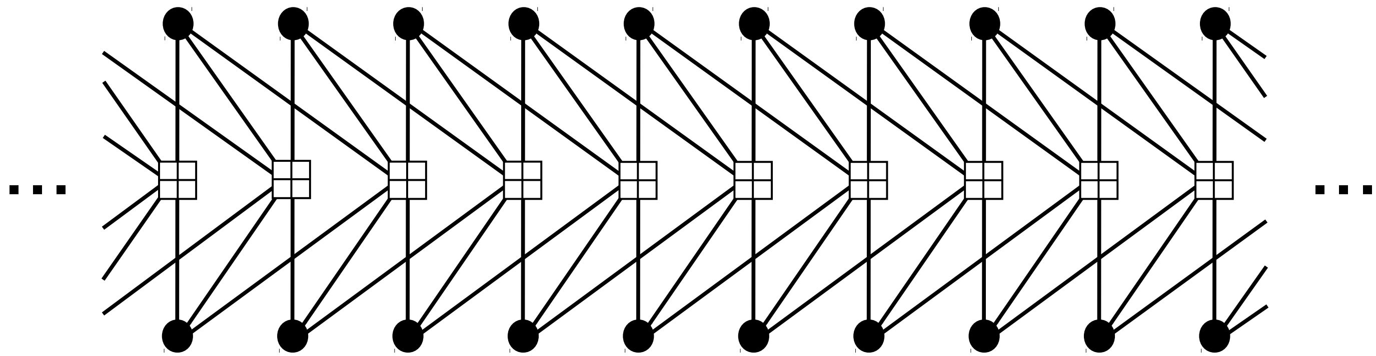

- The bi-infinite convolutional protograph corresponds to a bi-infinite **convolutional base matrix**:

$$\mathbf{B}_{[-\infty,\infty]} = \begin{bmatrix} \ddots & & \ddots & \ddots & & & & \\ & \mathbf{B}_{m_s} & \cdots & \mathbf{B}_1 & \mathbf{B}_0 & & & \\ & & \ddots & & \ddots & \ddots & & \\ & & & \mathbf{B}_{m_s} & \cdots & \mathbf{B}_1 & \mathbf{B}_0 & \\ & & & & \ddots & & \ddots & \ddots \end{bmatrix}$$

Rate:

$$R = \frac{b_v - b_c}{b_v}$$

Constraint length:

$$\nu_s = b_v(m_s + 1)$$

SC-LDPC Code Ensembles

- An ensemble of (3,6)-regular SC-LDPC codes can be created from the **convolutional protograph** by the graph lifting operation

$$\mathbf{B}_{[-\infty,\infty]} = \begin{bmatrix} \ddots & \ddots & \ddots & & & \\ \mathbf{B}_2 & \mathbf{B}_1 & \mathbf{B}_0 & & & \\ & \mathbf{B}_2 & \mathbf{B}_1 & \mathbf{B}_0 & & \\ & & \mathbf{B}_2 & \mathbf{B}_1 & \mathbf{B}_0 & \\ & & & \ddots & \ddots & \ddots \end{bmatrix}$$

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$$\mathbf{B}_i = [\,1 \quad 1\,]$$
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Graph lifting: $\Pi_{i,j}$ is an $M \times M$ permutation matrix

$$\nu_s = M b_v (m_s + 1) = 6M$$

$$\mathbf{H}_{\text{cc}} = \begin{bmatrix} \ddots & & \ddots & & \ddots & & & & & \\ \Pi_{5,t} & \Pi_{4,t} & \Pi_{3,t} & \Pi_{2,t} & \Pi_{1,t} & \Pi_{0,t} & & & & \\ & & \Pi_{5,t+1} & \Pi_{4,t+1} & \Pi_{3,t+1} & \Pi_{2,t+1} & \Pi_{1,t+1} & \Pi_{0,t+1} & & \\ & & & & \Pi_{5,t+2} & \Pi_{4,t+2} & \Pi_{3,t+2} & \Pi_{2,t+2} & \Pi_{1,t+2} & \Pi_{0,t+2} \\ & & & & & \ddots & & \ddots & & \ddots \end{bmatrix}$$

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- If each permutation matrix $\Pi_{i,j}$ is **circulant**, the codes are **quasi-cyclic**

Terminated Spatially Coupled Codes

- Consider **terminating** $\mathbf{B}_{[-\infty,\infty]}$ to a (block code) **base matrix** of length Lb_v:

$$\mathbf{B}_{[0,L-1]} = \begin{bmatrix} \mathbf{B}_0 & & \\ \vdots & \ddots & \\ \mathbf{B}_{m_s} & & \mathbf{B}_0 \\ & \ddots & \vdots \\ & & \mathbf{B}_{m_s} \end{bmatrix}_{(L+m_s)b_c \times Lb_v}$$

Code rate:

$$R_L = \frac{Lb_v - (L+m_s)b_c}{Lb_v}.$$

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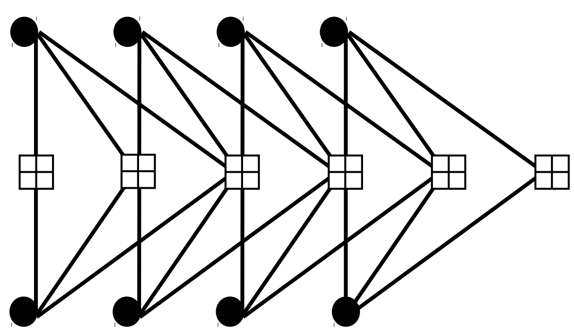
$$\mathbf{B}_{[0,3]} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$



(**check node degrees lower at the ends**)

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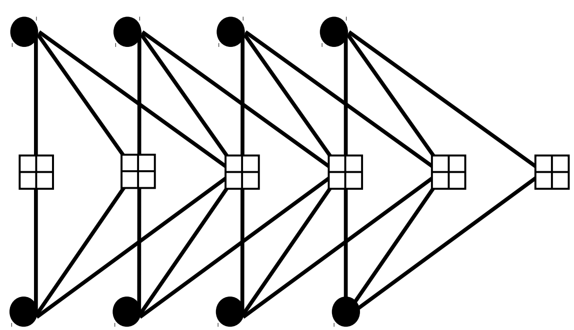

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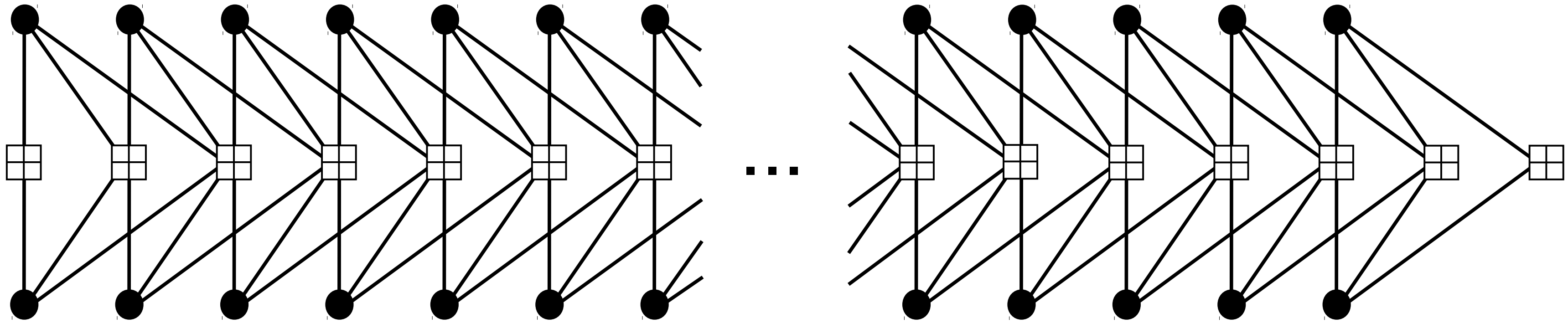
- Codes can be **lifted** to **different lengths and rates** by varying M and L.

Thresholds of SC-LDPC Codes

- Variable nodes all have the **same degree** as the block code.
- Check nodes with **lower degrees** (at the ends) improve the BP decoder.



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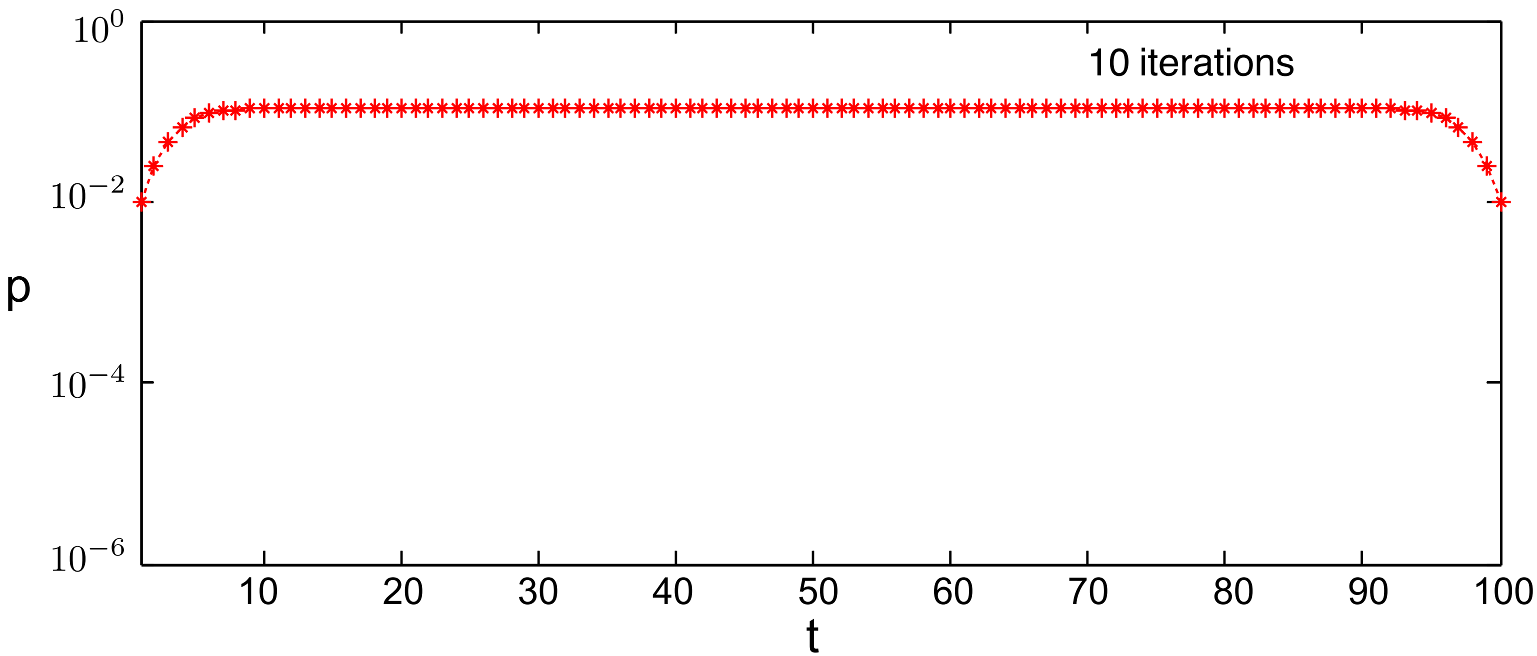

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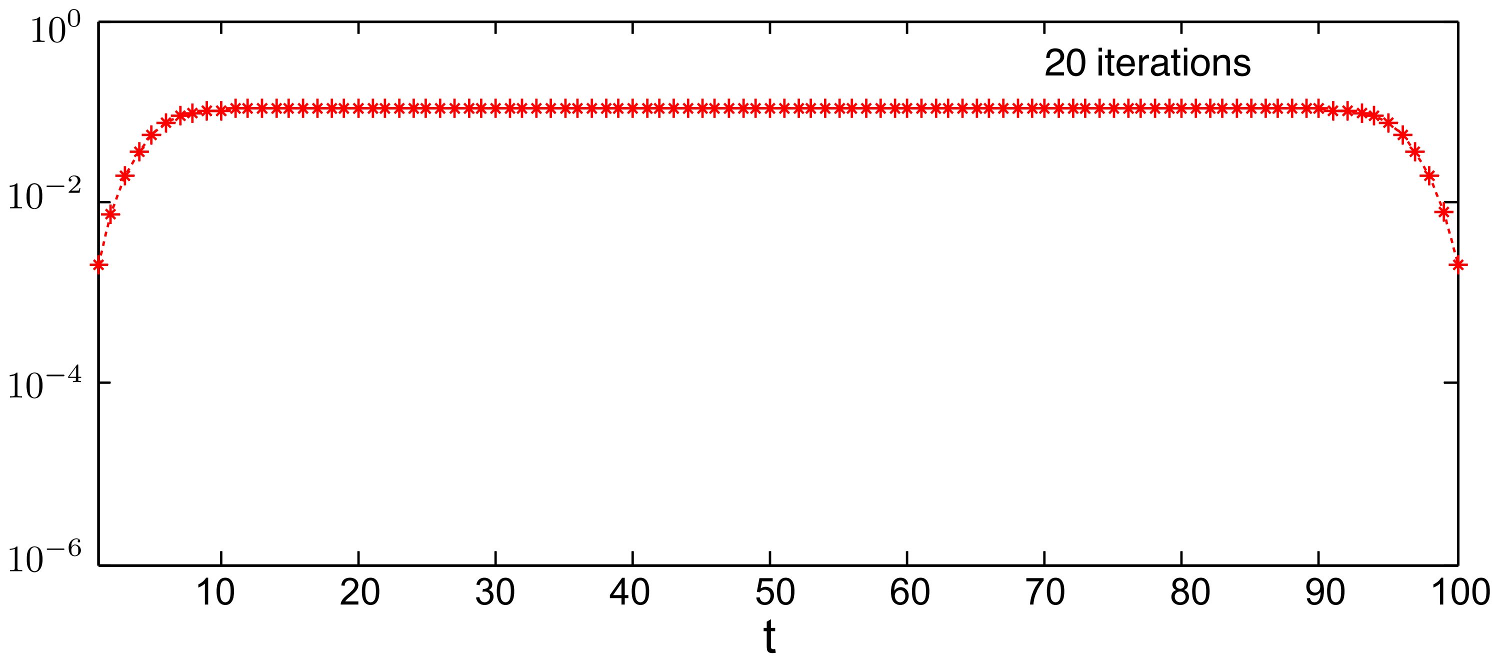

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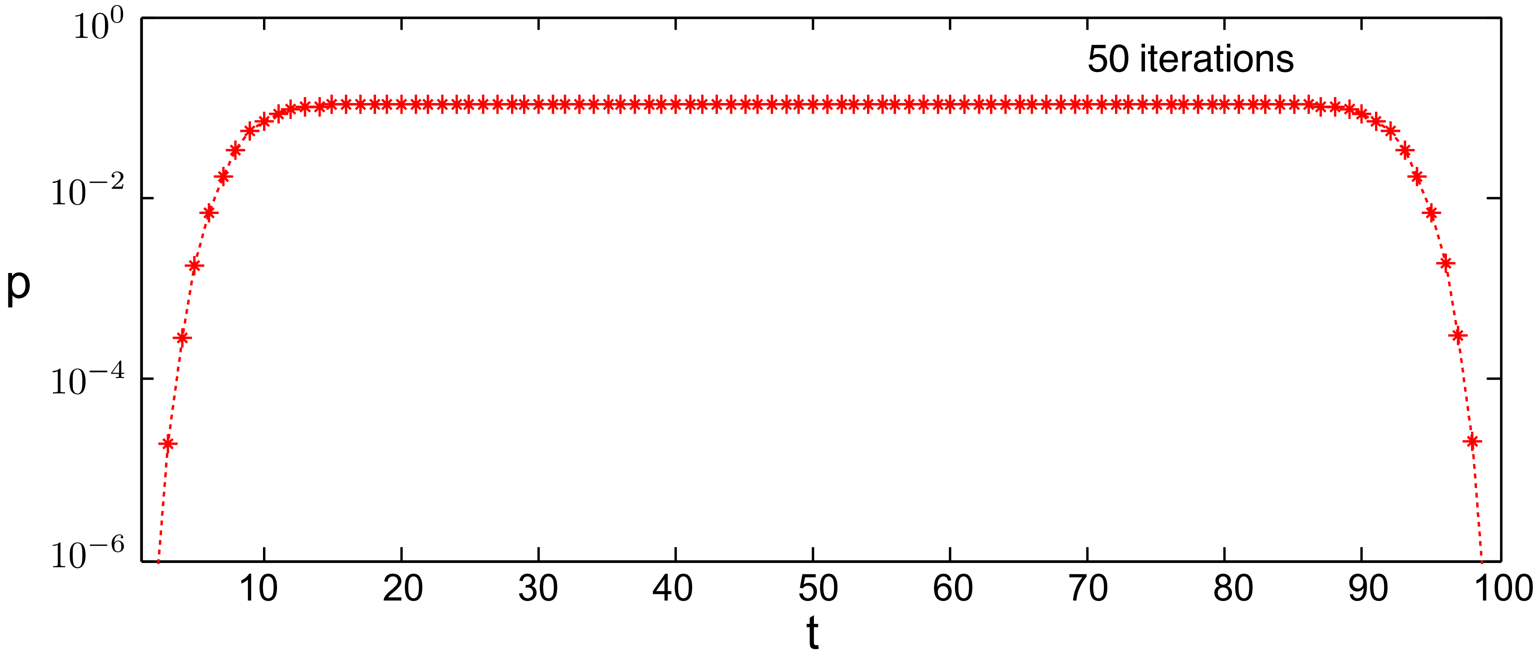

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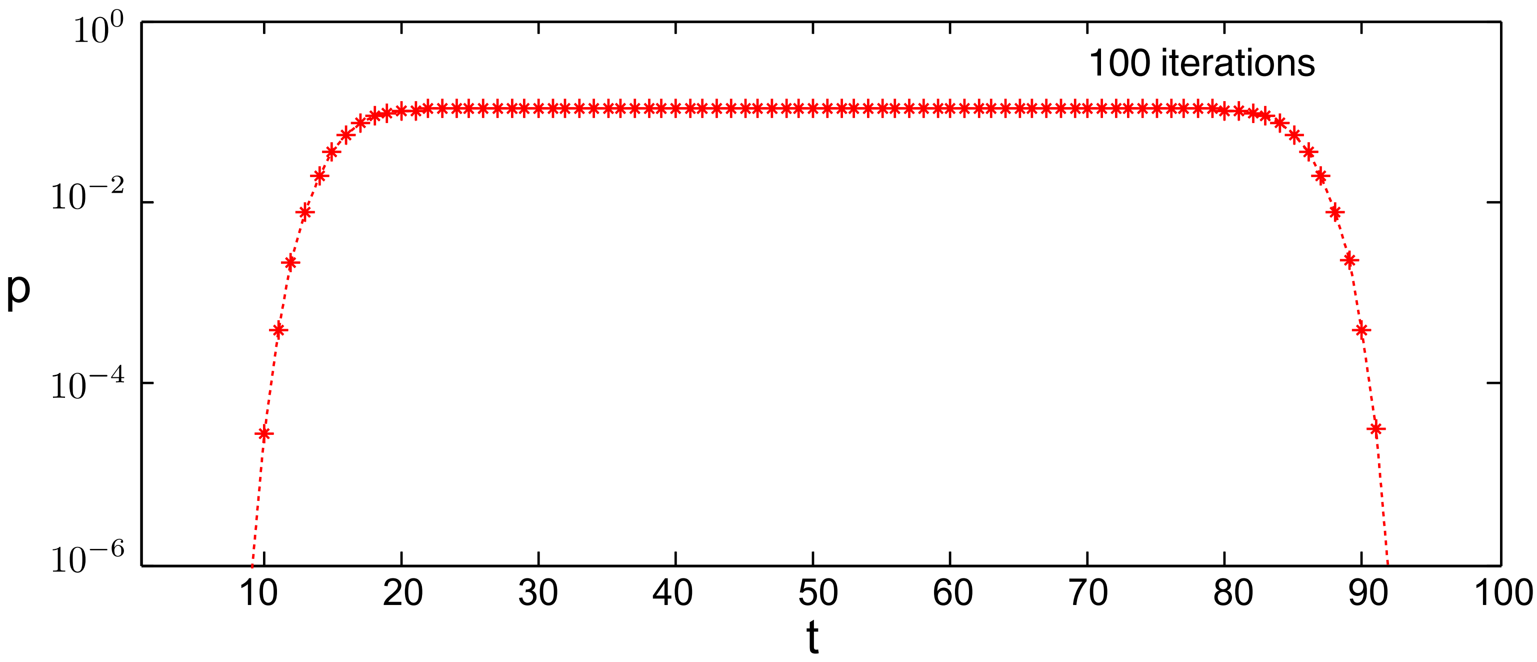

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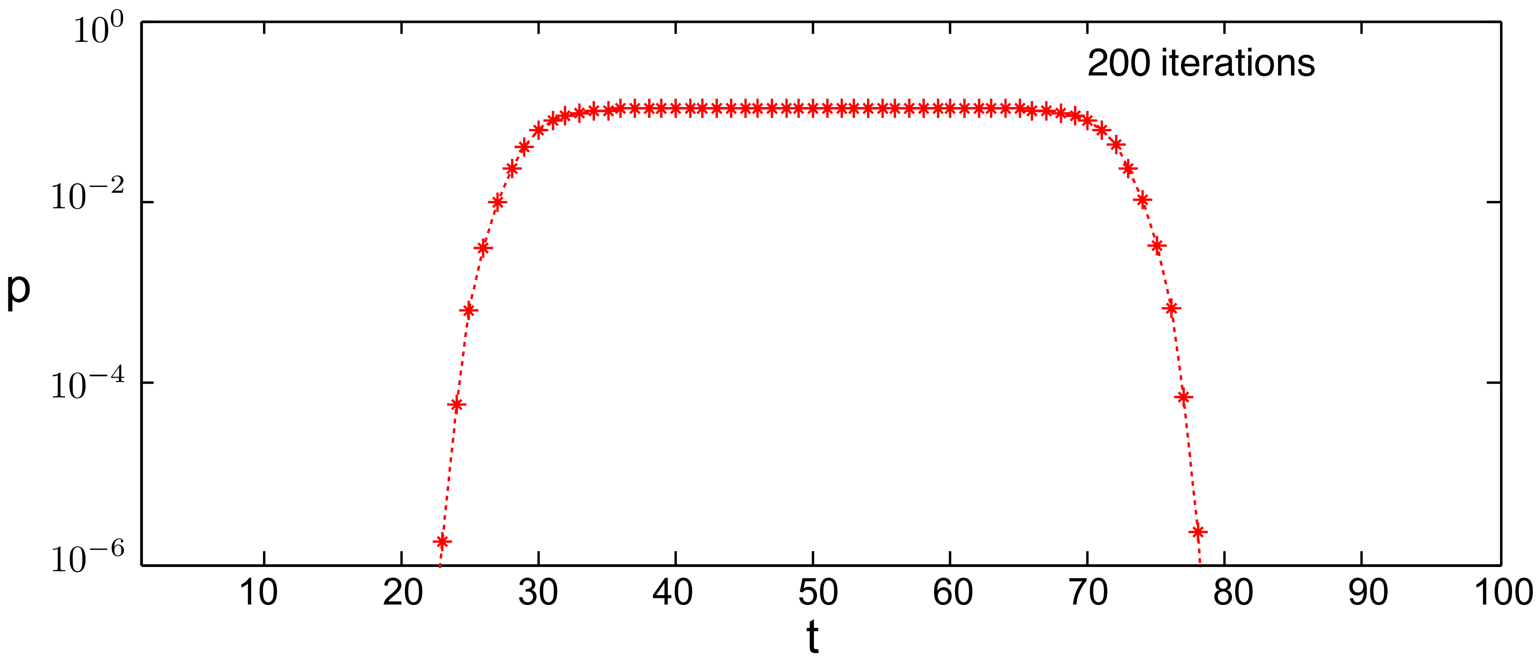

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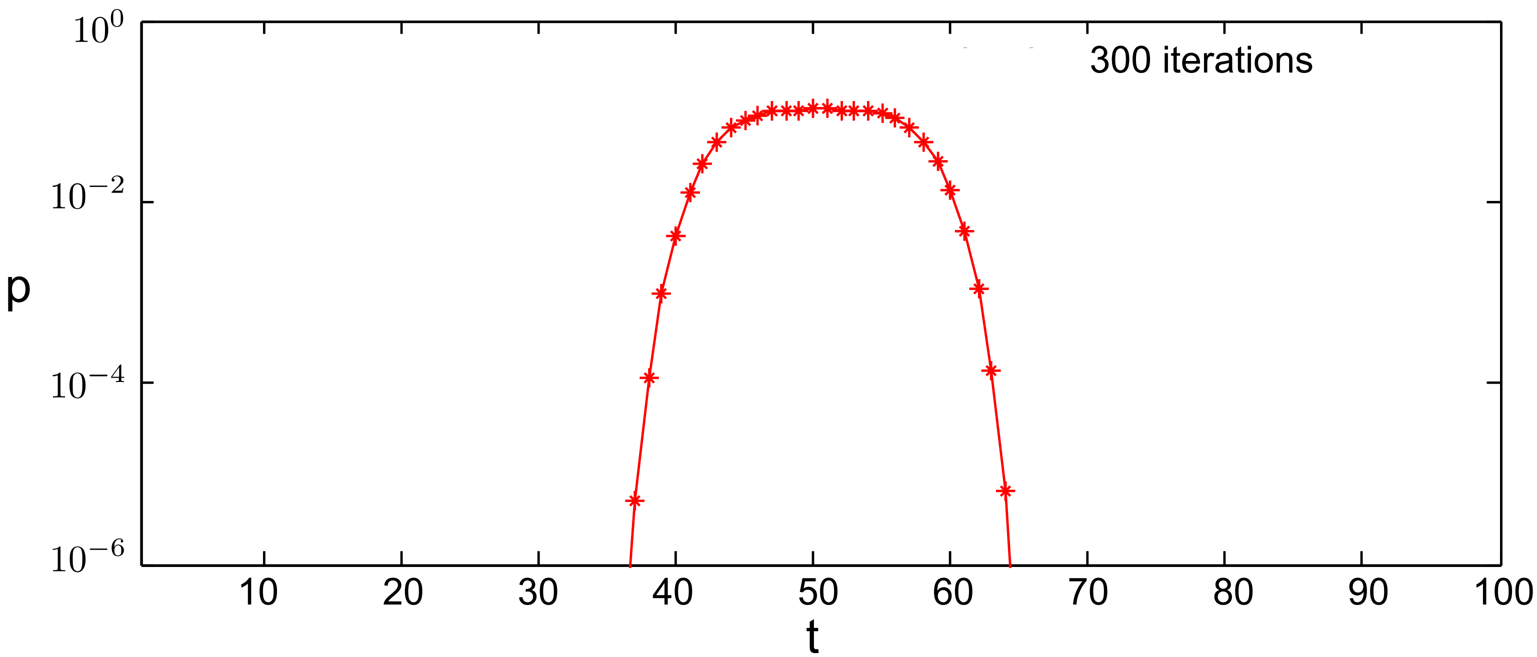

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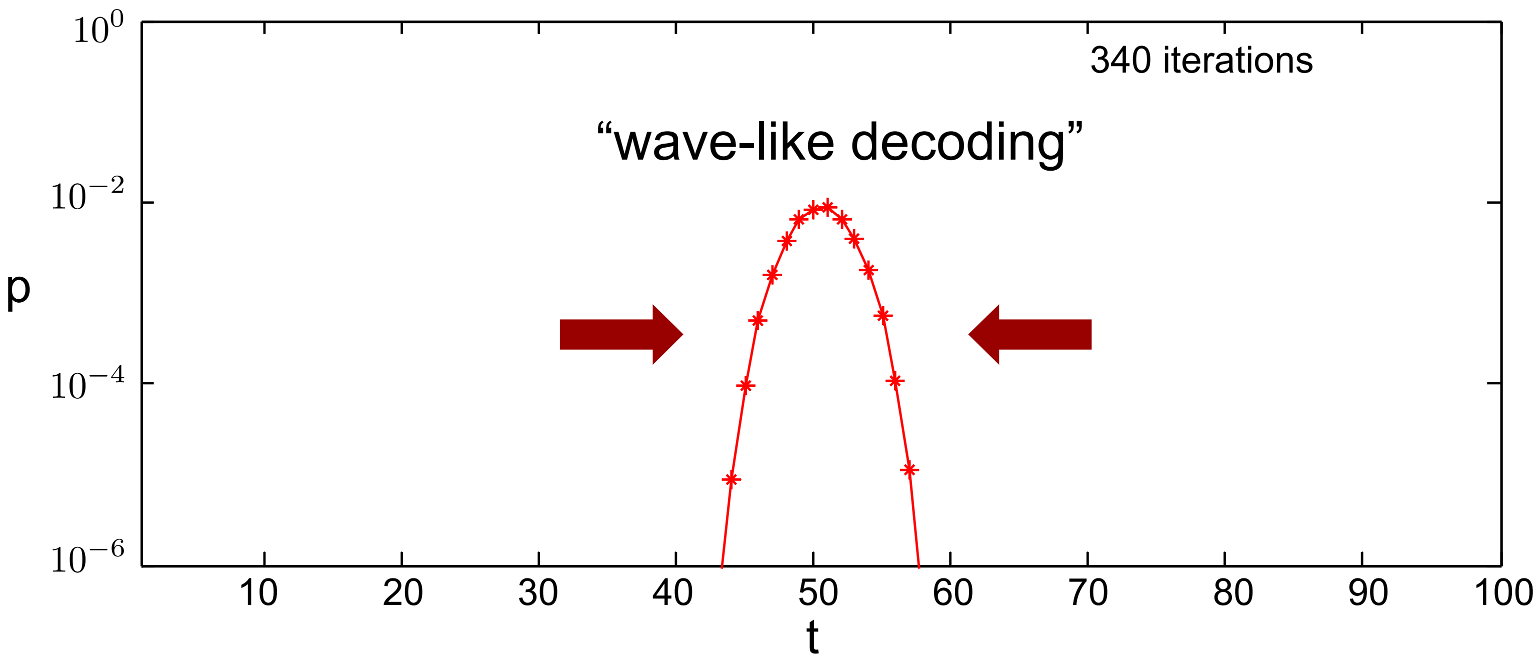

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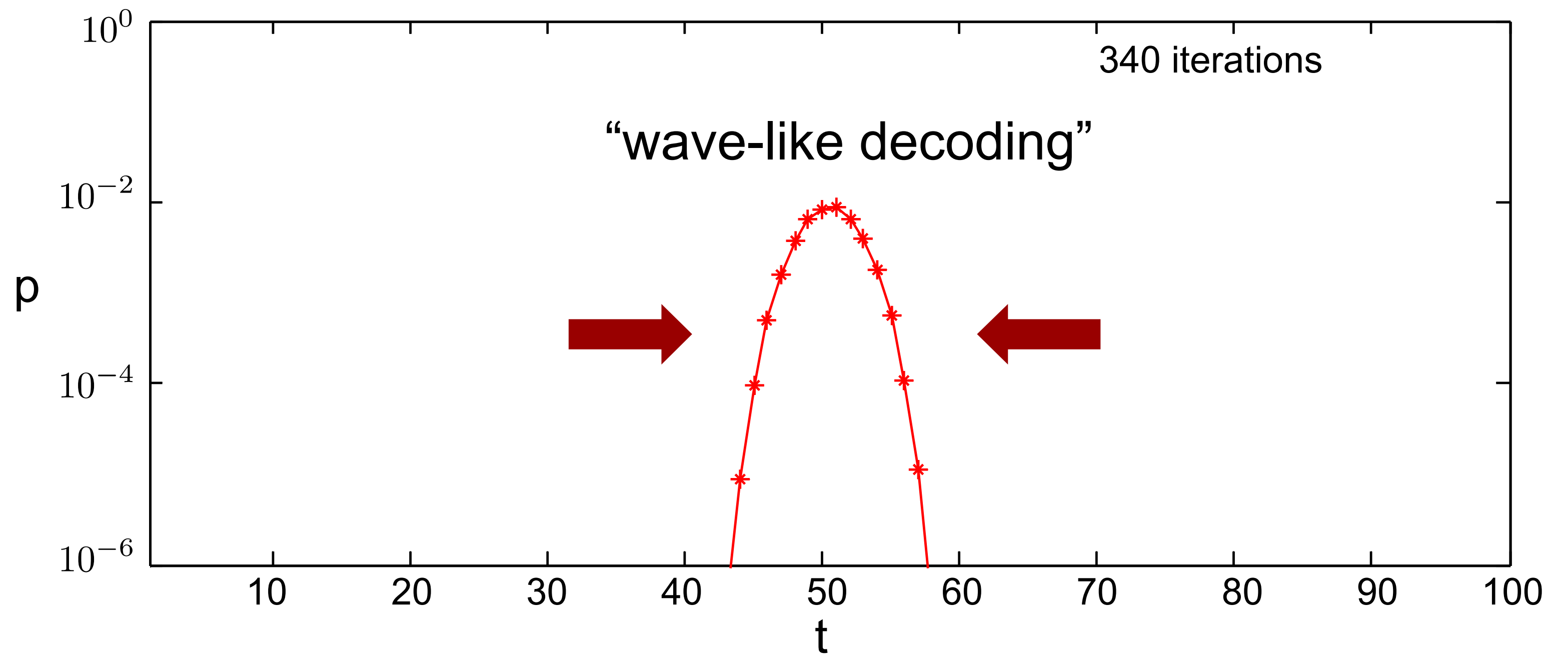
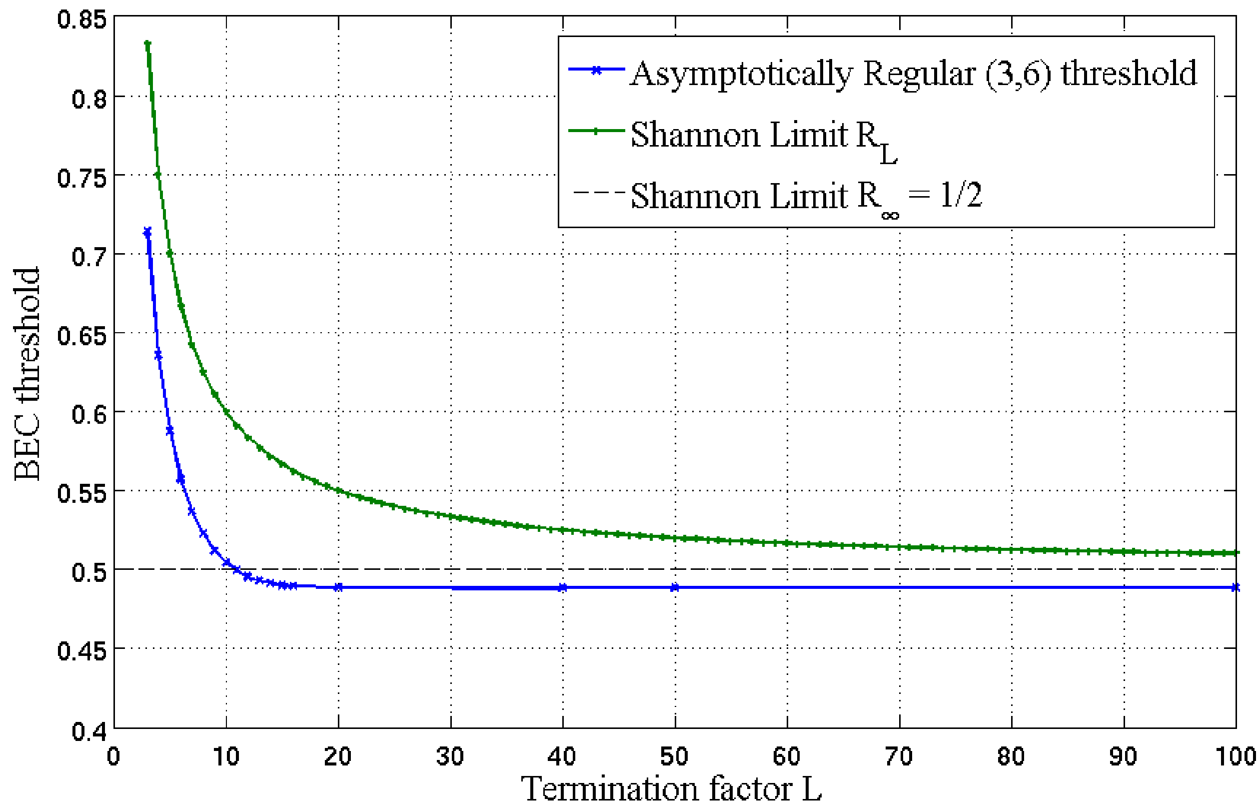


- Note: the **fraction** of **lower degree** nodes tends to zero as $L \to \infty$, i.e., the codes are **asymptotically regular**.

Thresholds of SC-LDPC Codes

- **Density evolution** can be applied to the protograph-based ensembles with $M \to \infty$ [Sridharan et al. '04]:

Example: BEC



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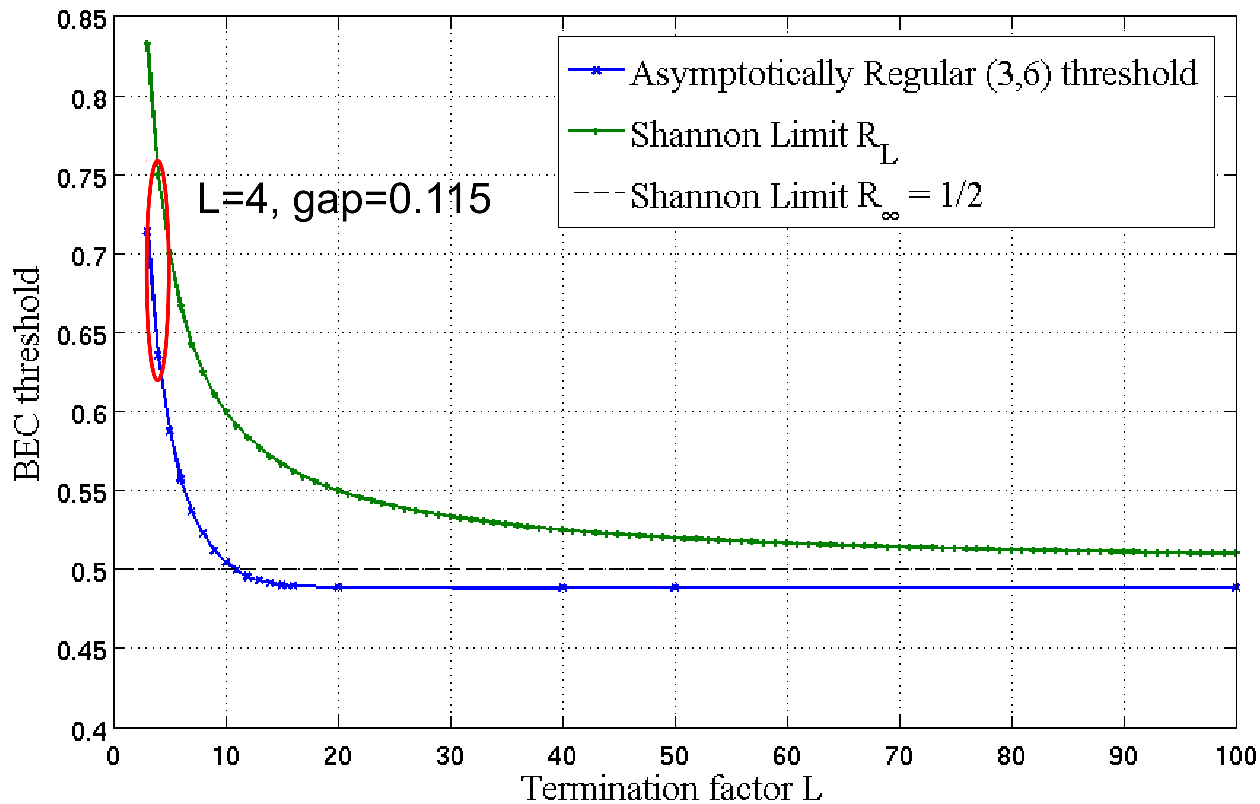
$L = 4$, $R = 1/4$

$\varepsilon^* = 0.635$, $\varepsilon_{\mathrm{Sh}} = 0.75$



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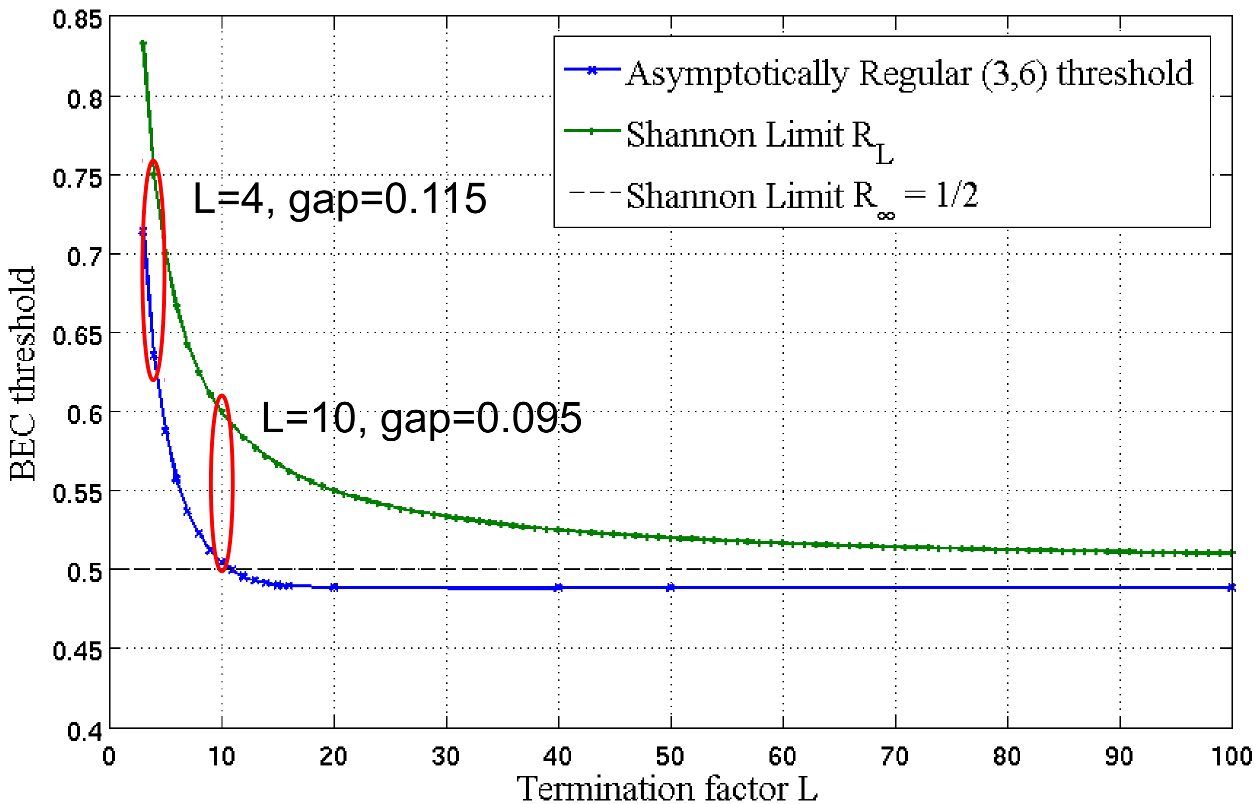

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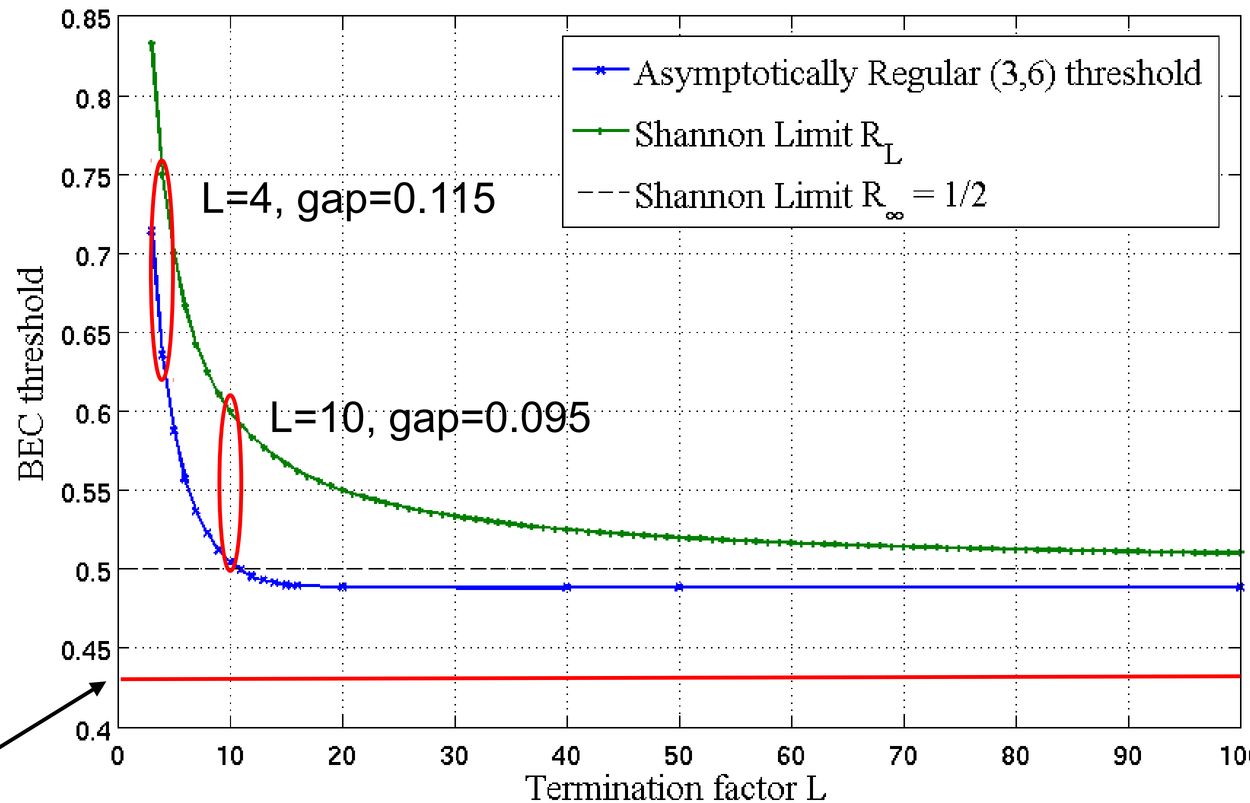
⋮

$L \to \infty$, $R \to 1/2$
$\varepsilon^* = 0.488$, $\varepsilon_{\mathrm{Sh}} = 0.5$

(3,6)-regular block code:

$\varepsilon^* = 0.429$



Thresholds of SC-LDPC Codes

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Iterative decoding thresholds (protograph-based ensembles)

BEC

(J,K)	ϵ^*_{SC}	$\epsilon^*_{\mathrm{blk}}$
(3,6)	0.488	0.429
(4,8)	0.497	0.383
(5,10)	0.499	0.341

AWGN

(J,K)	E_b/N_o SC	E_b/N_o blk
(3,6)	0.46 dB	1.11 dB
(4,8)	0.26 dB	1.61 dB
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- We observe a **significant improvement** in the thresholds of SC-LDPC codes compared to the associated LDPC block codes (LDPC-BCs) due to the lower degree check nodes at the ends of the graph and the wave-like decoding.

[LSCZ10] M. Lentmaier, A. Sridharan, D. J. Costello, Jr., and K.Sh. Zigangirov, "Iterative decoding threshold analysis for LDPC convolutional codes," *IEEE Trans. Inf. Theory*, 56:10, Oct. 2010.

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- In contrast to LDPC-BCs, the iterative decoding thresholds of SC-LDPC codes **improve** as the graph density increases.

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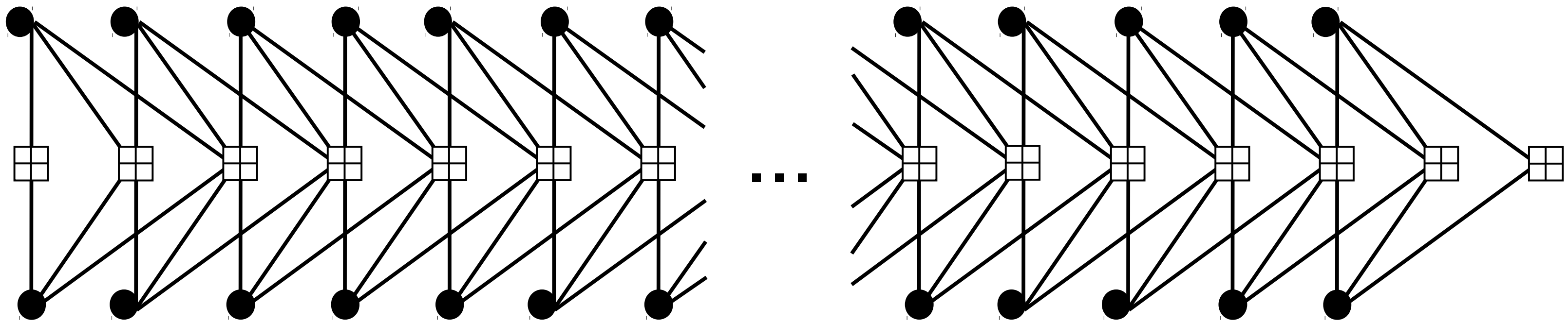
Why are SC-LDPC Codes Better?

- When symbols are perfectly known (BEC), all adjacent edges can be removed from the Tanner graph.



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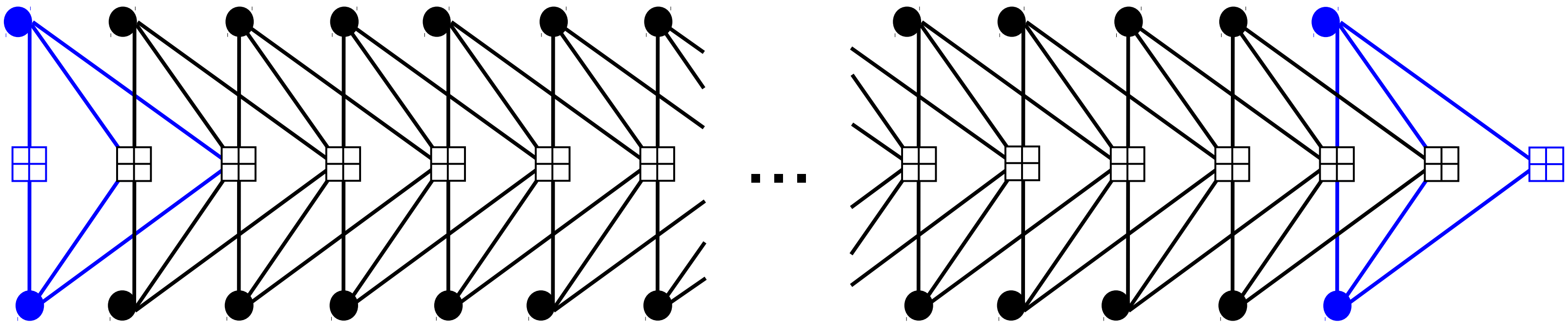

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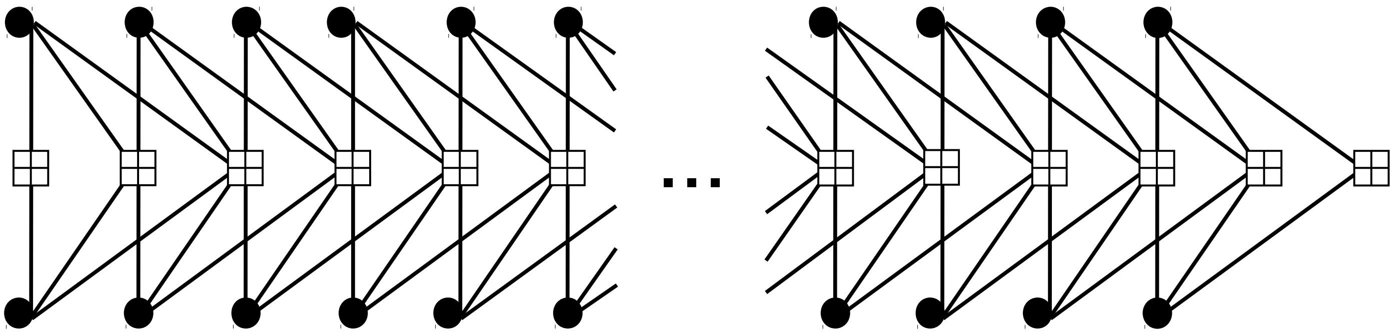

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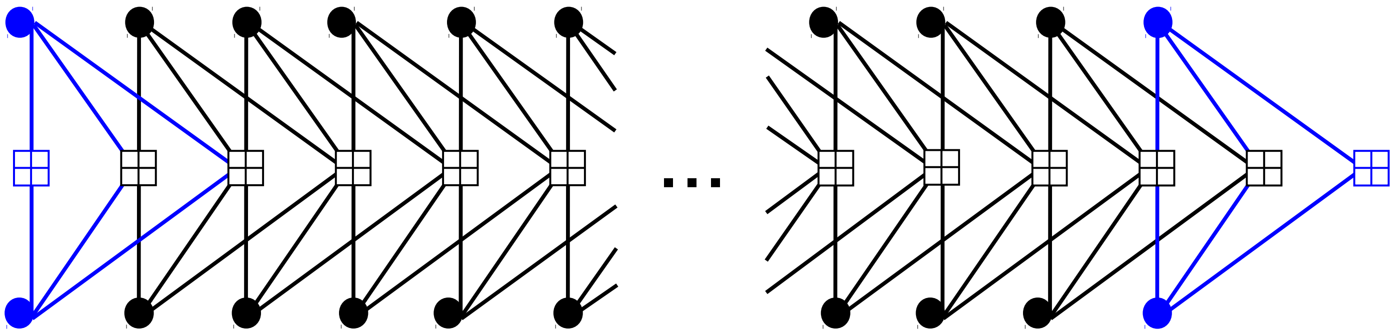

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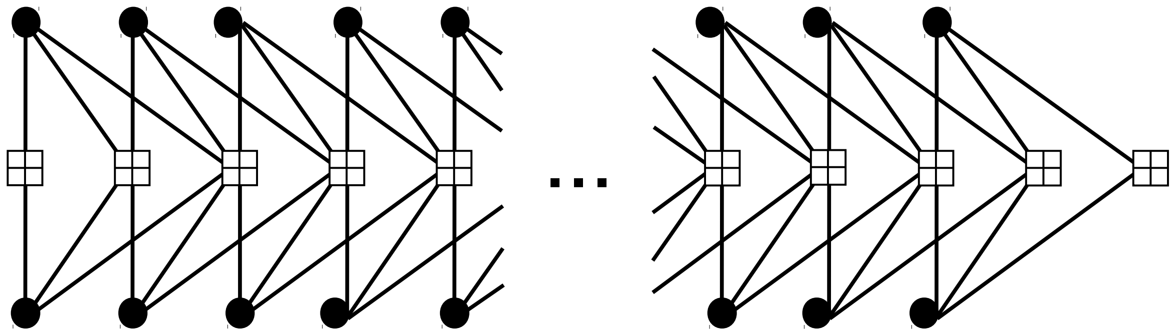

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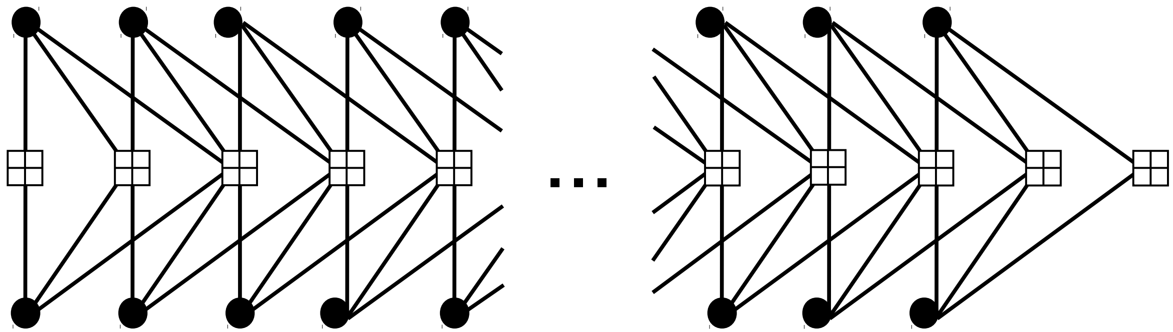

- The **threshold saturates** (converges) to a fixed value numerically indistinguishable from the **maximum a posteriori** (MAP) threshold of the (J, K)-regular LDPC-BC ensemble as $L \to \infty$ [LSCZ10].

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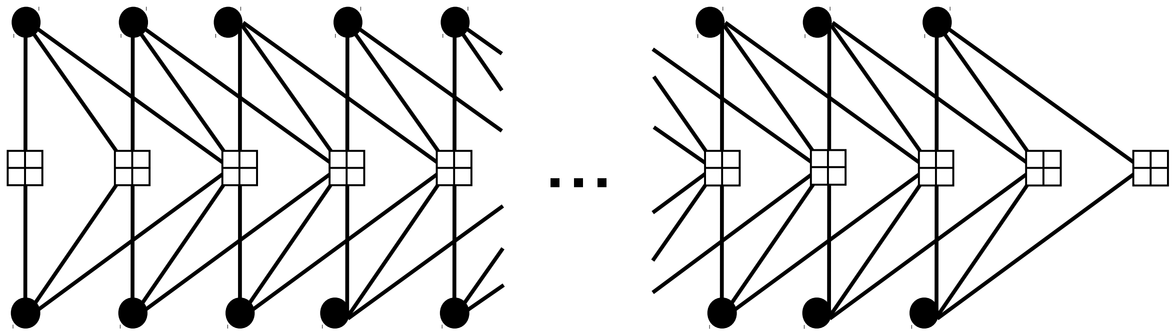

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- For a more random-like ensemble, this has been proven analytically, first for the BEC [KRU11], then for all BMS channels [KRU13].

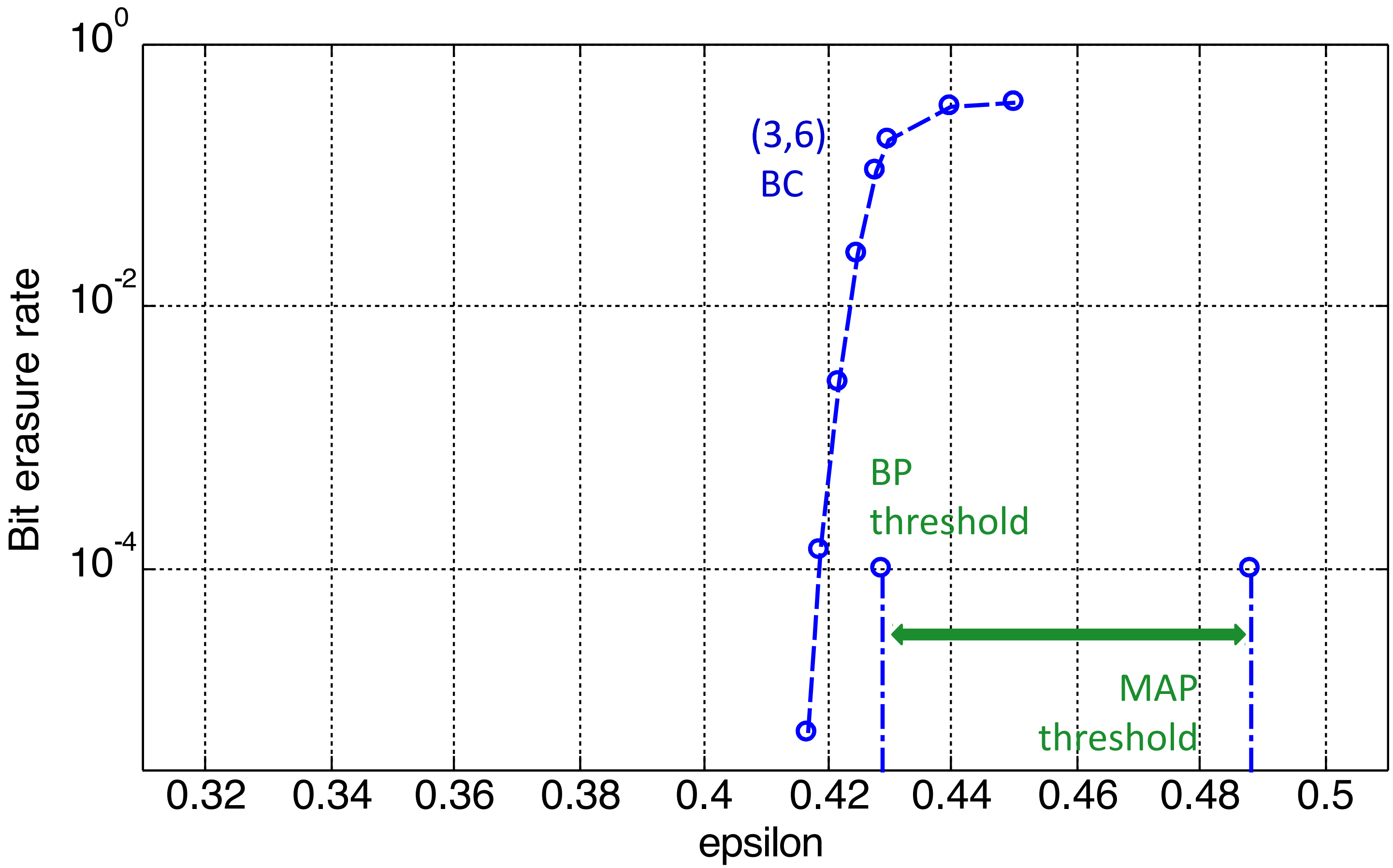
[LSCZ10] M. Lentmaier, A. Sridharan, D. J. Costello, Jr., and K.Sh. Zigangirov, "Iterative decoding threshold analysis for LDPC convolutional codes," *IEEE Trans. Inf. Theory*, 56:10, Oct. 2010.

[KRU11] S. Kudekar, T. J. Richardson and R. Urbanke, "Threshold saturation via spatial coupling: why convolutional LDPC ensembles perform so well over the BEC", *IEEE Trans. on Inf. Theory*, 57:2, 2011

[KRU13] S. Kudekar, T. J. Richardson and R. Urbanke, "Spatially coupled ensembles universally achieve capacity under belief propagation", *IEEE Trans. on Inf. Theory*, 59:12, 2013.

Threshold Saturation (BEC)

BP = iterative (suboptimal) decoding threshold
MAP = (optimal) maximum a posteriori threshold



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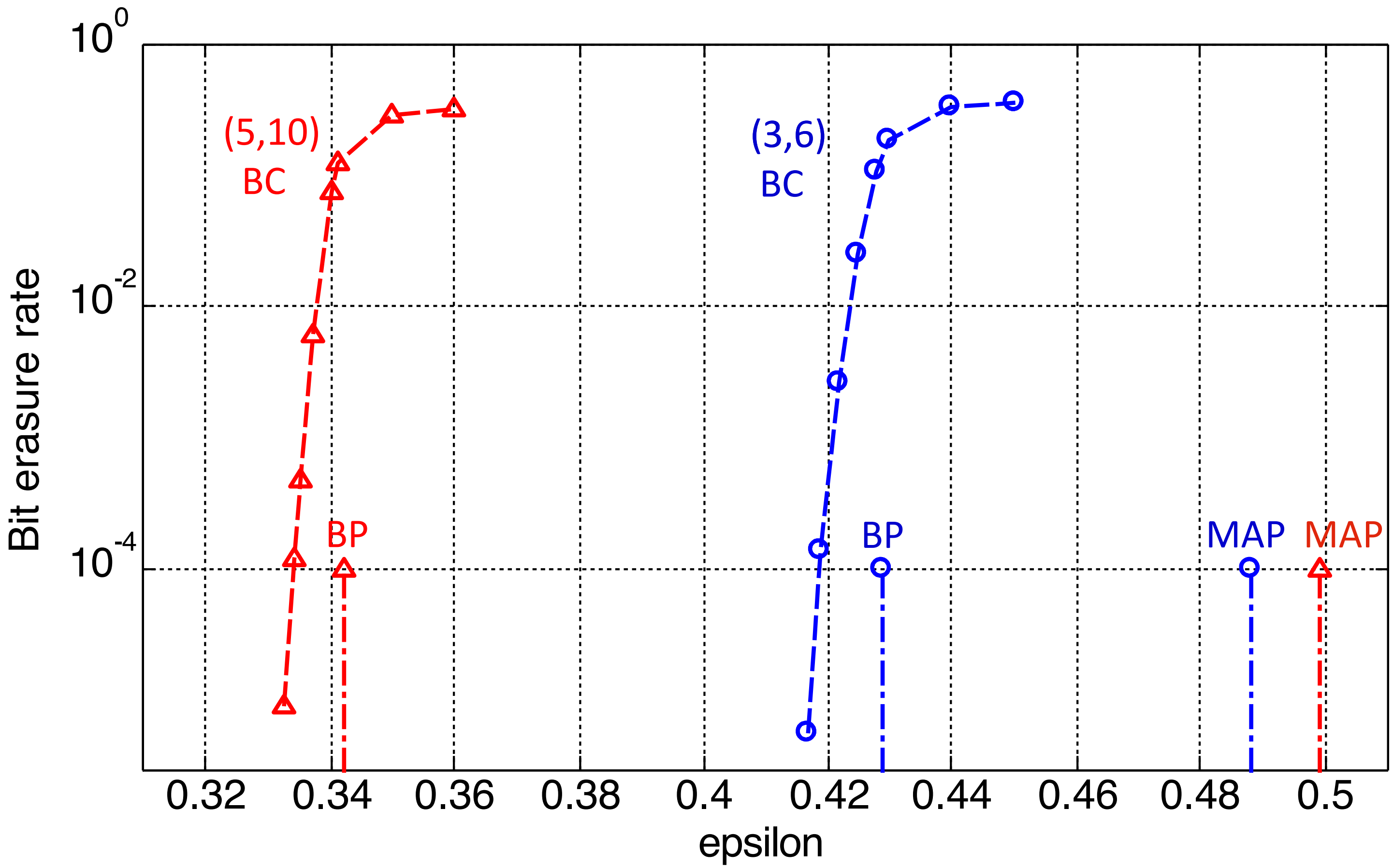

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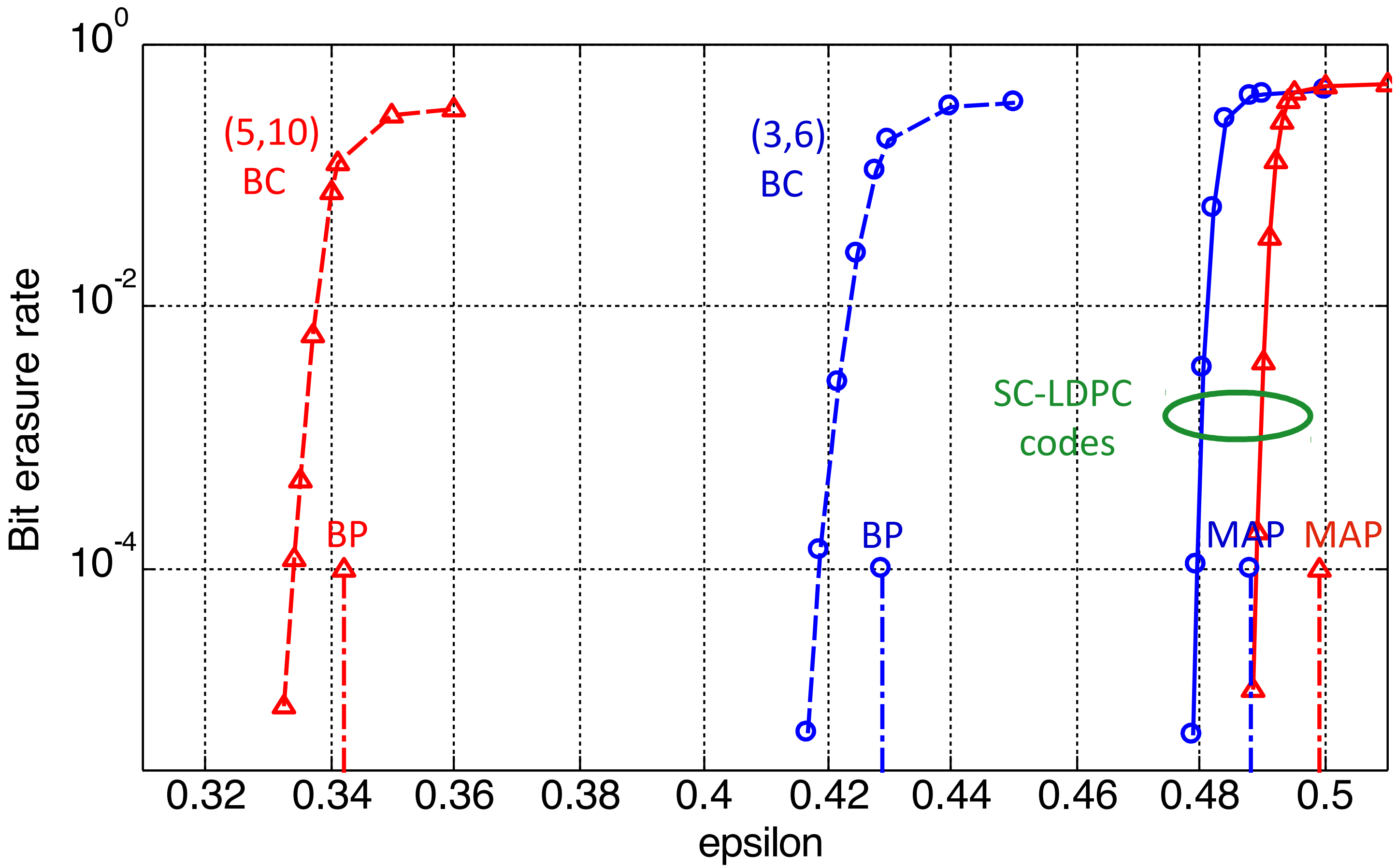

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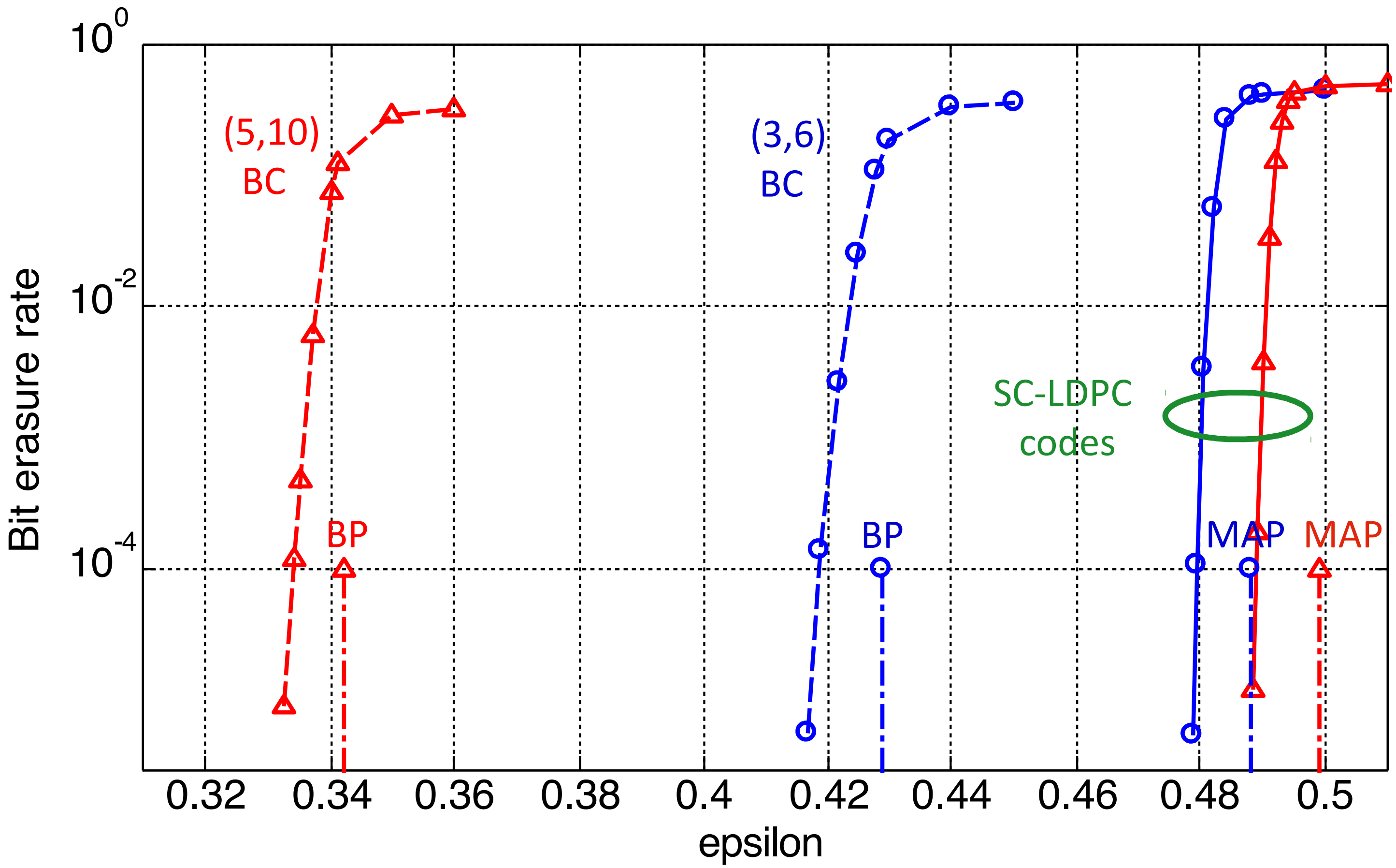

➡ **optimal** decoding performance with a **suboptimal** iterative algorithm!

Threshold Saturation (AWGNC)

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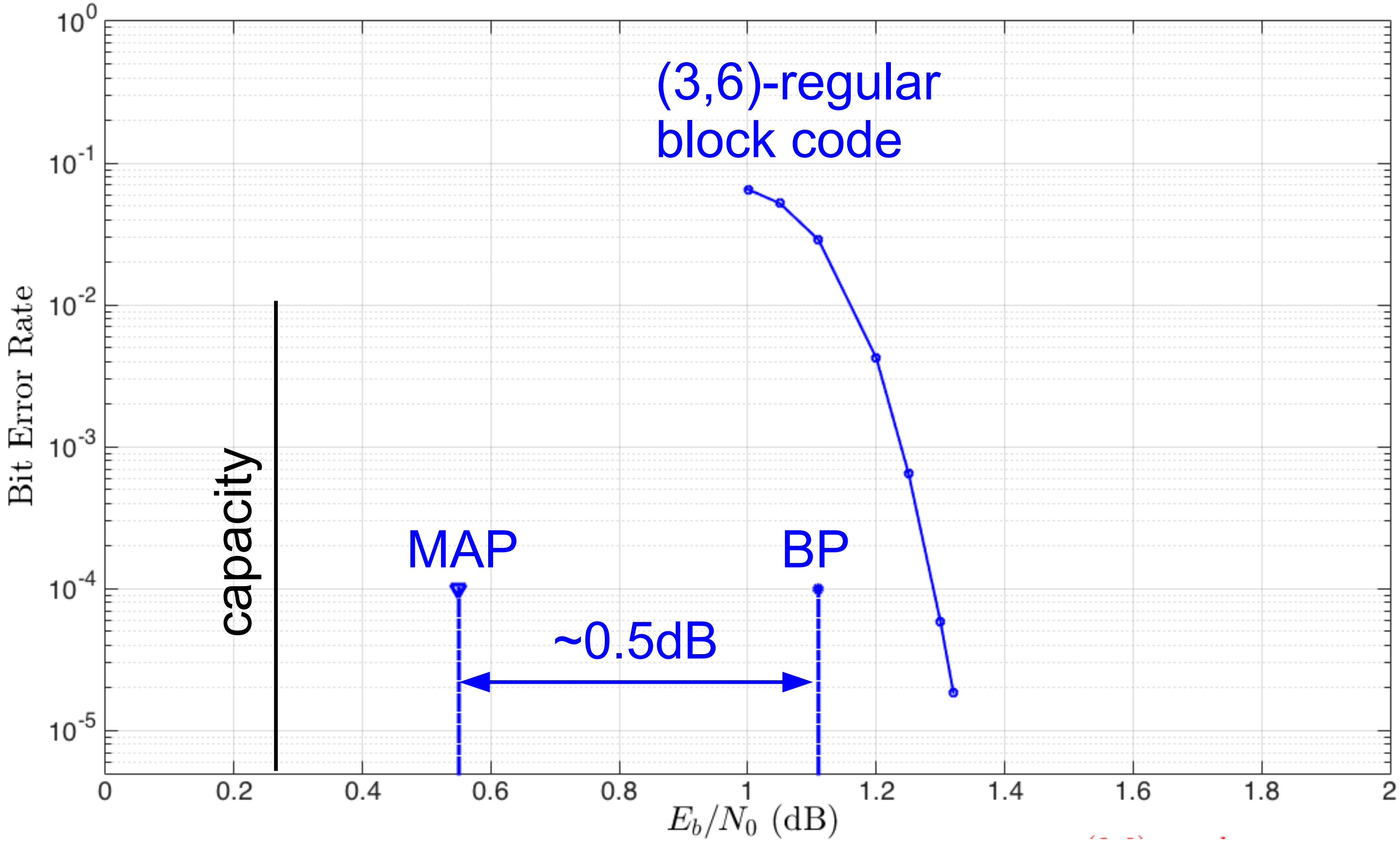

Threshold Saturation (AWGNC)

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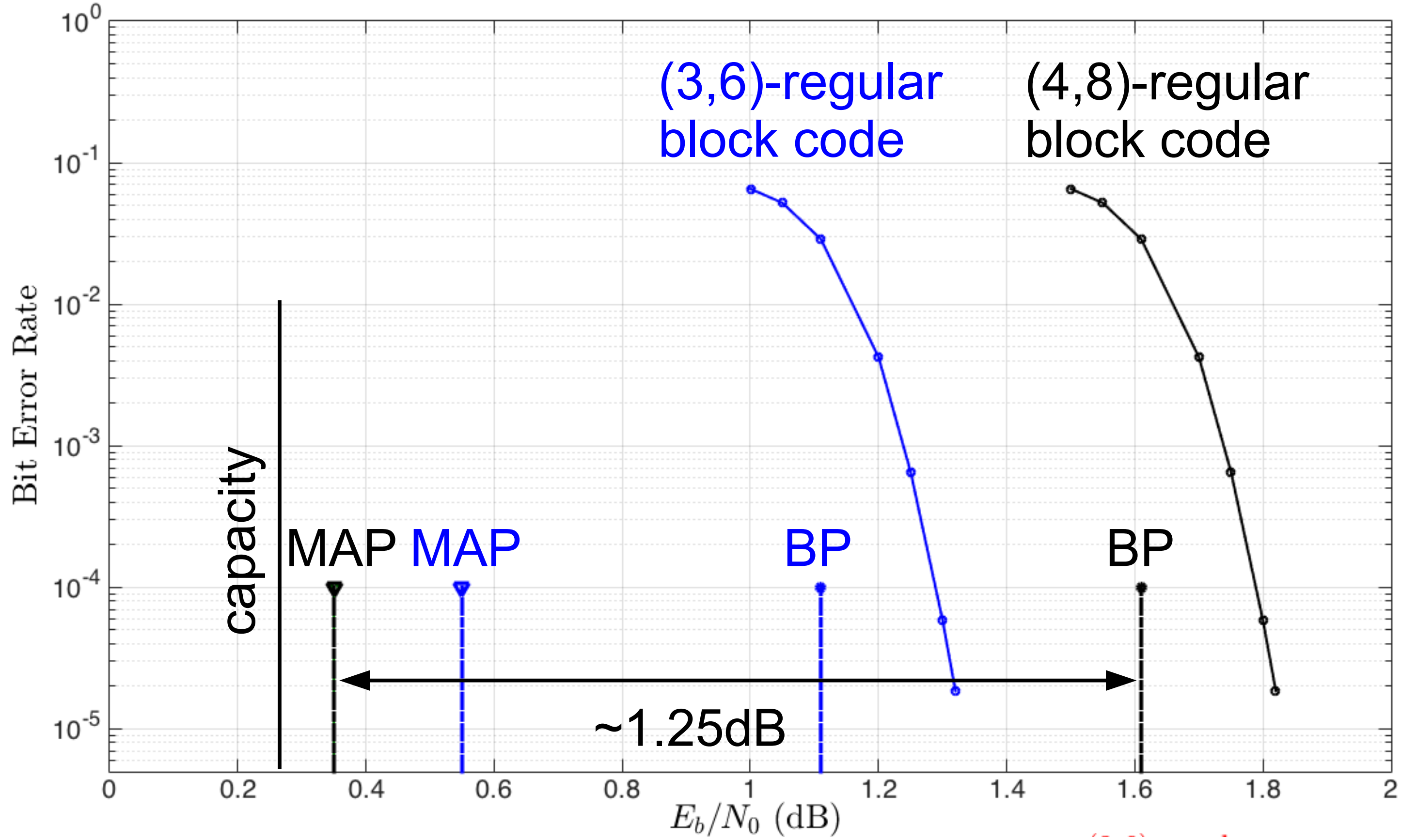

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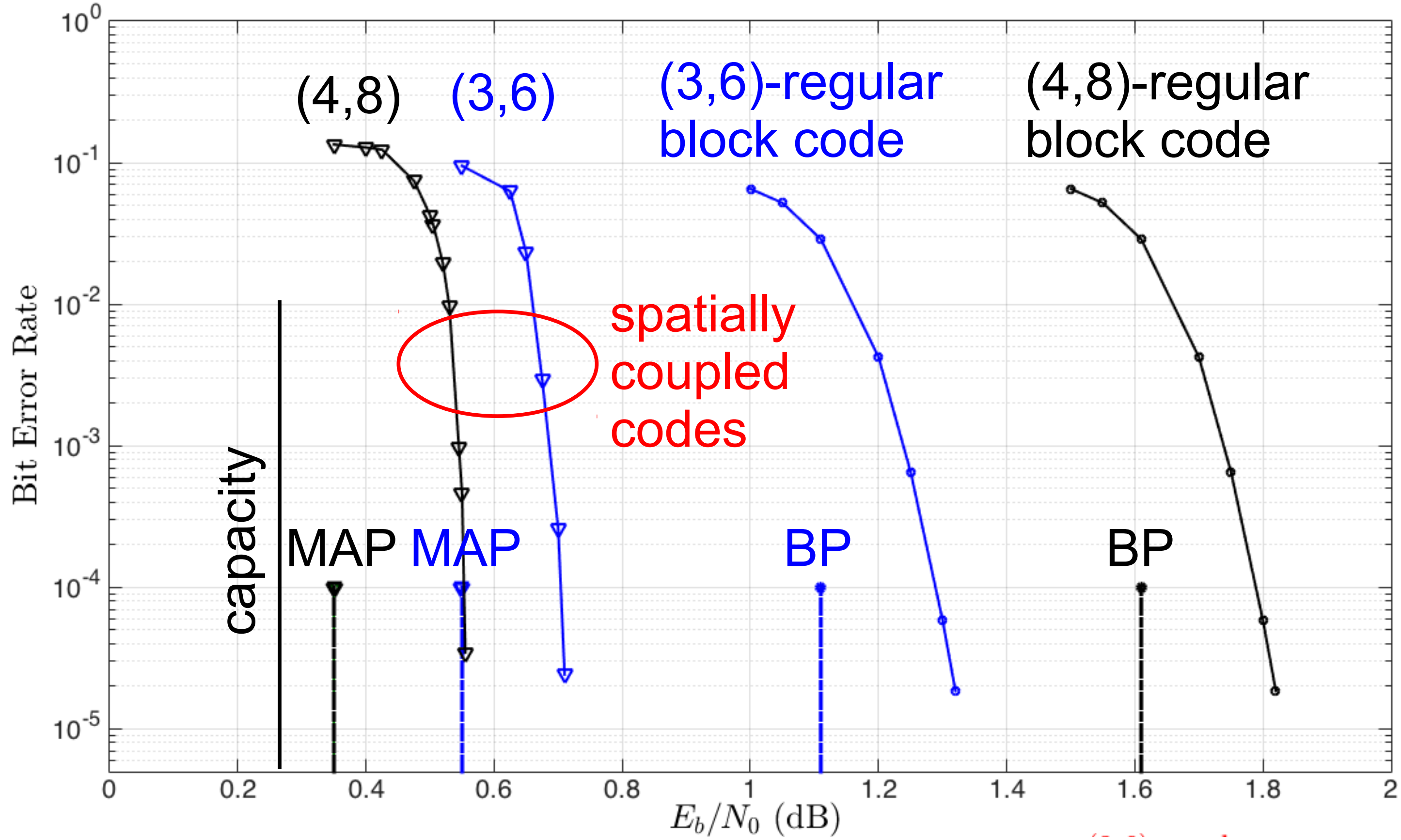

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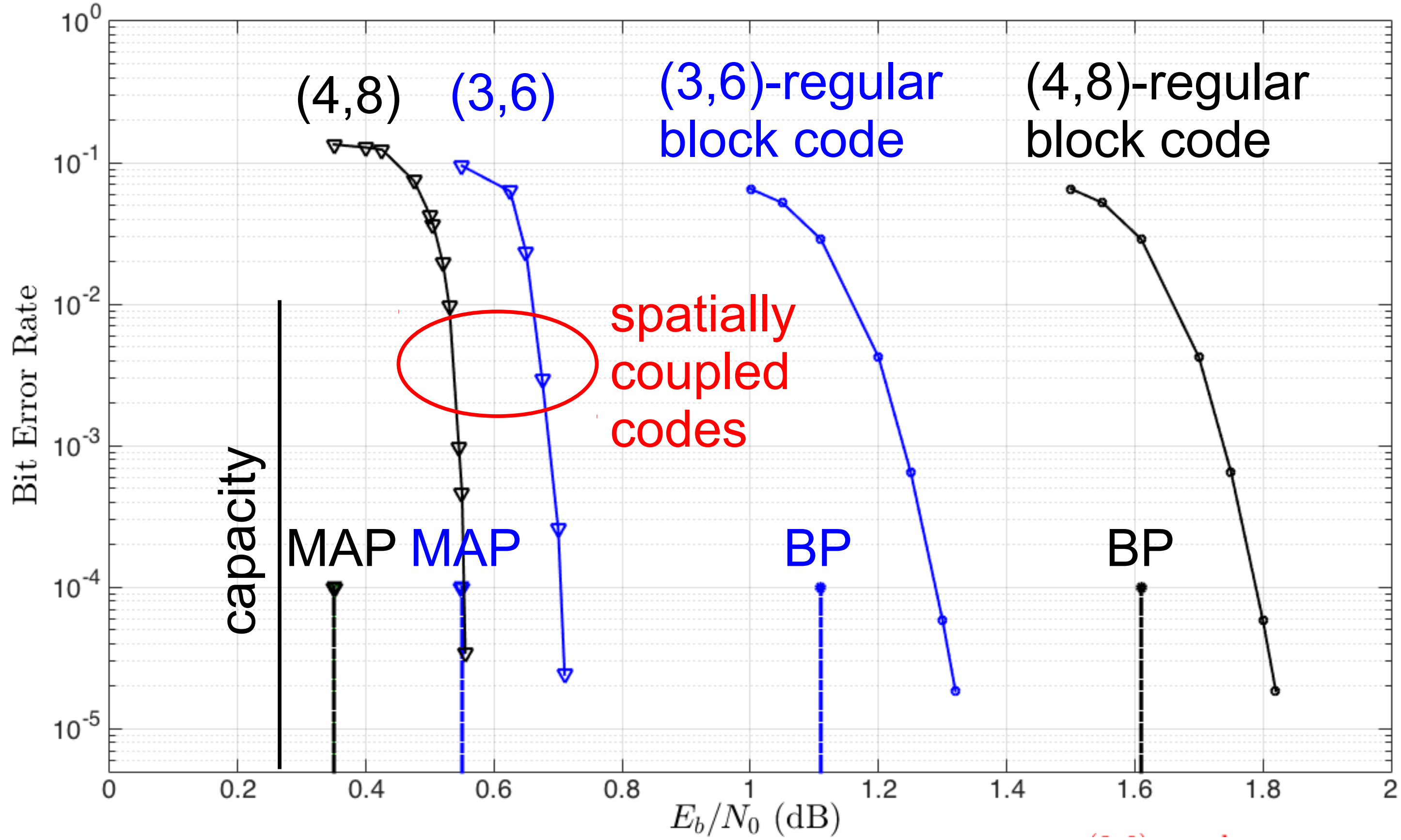

➡ **optimal** decoding performance with a **suboptimal** iterative algorithm!

BEC Thresholds vs Distance Growth

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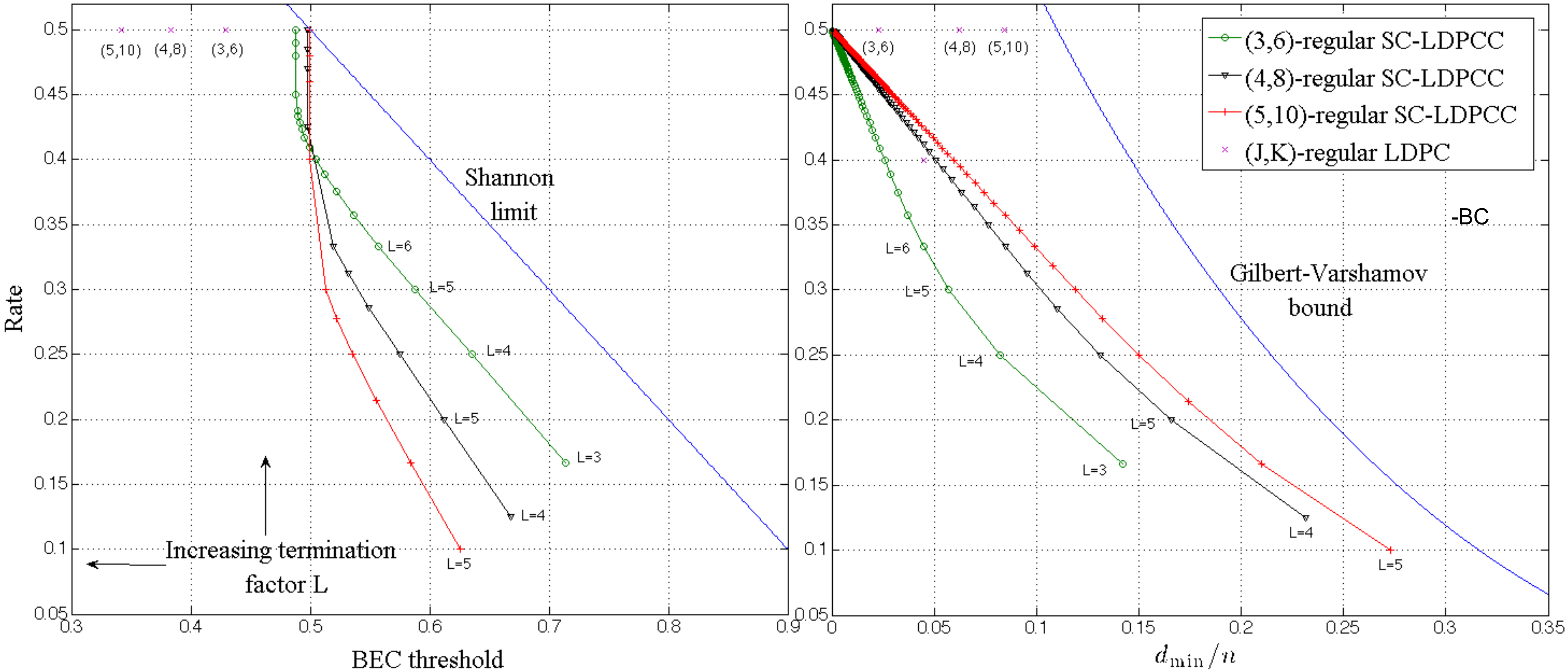
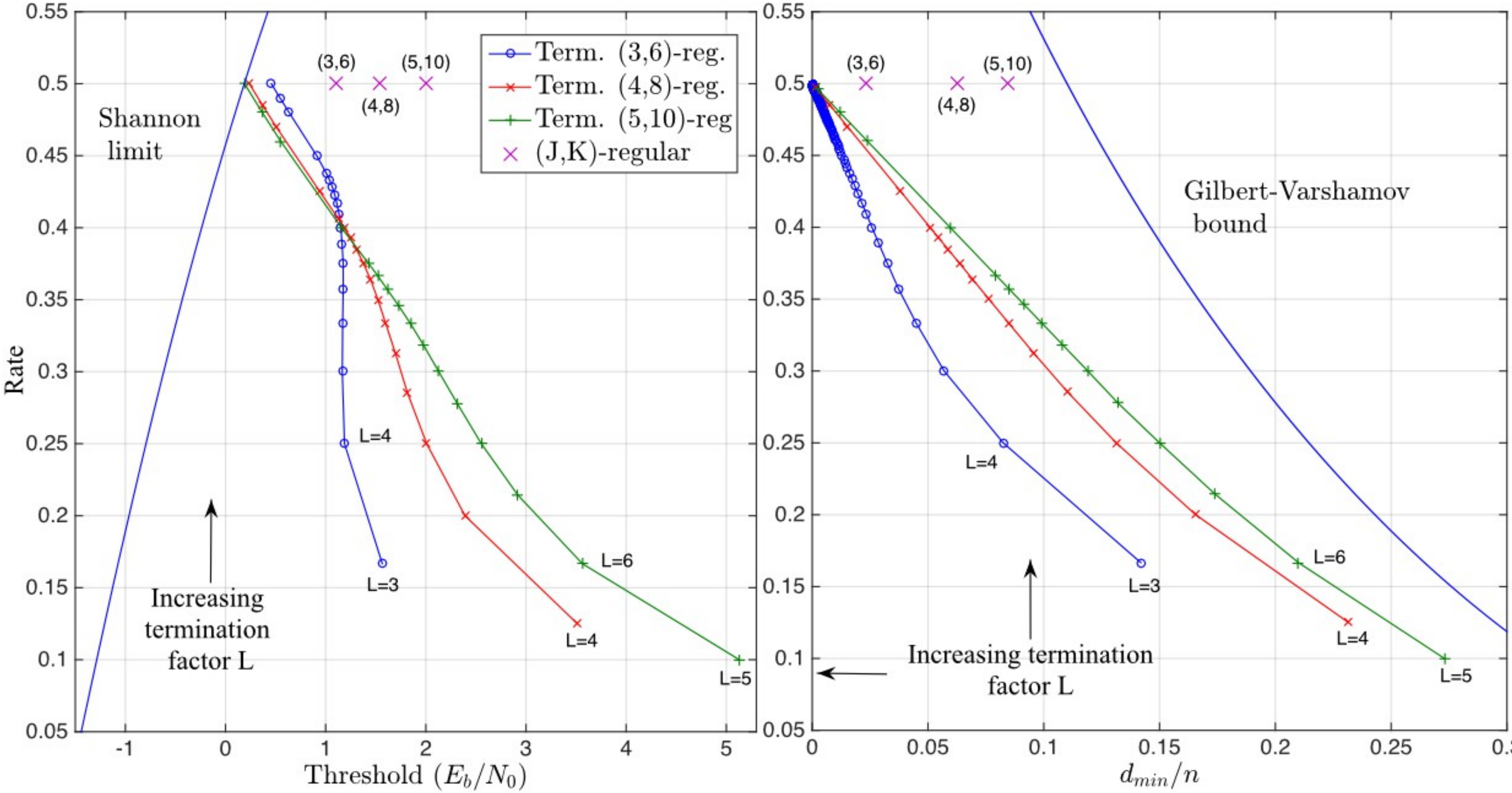
- By increasing J and K, we obtain **capacity achieving** (J,K)-regular SC-LDPC code ensembles with linear minimum distance growth.



- (J,K)-regular SC-LDPC codes combine the best features of irregular and regular LDPC-BCs, i.e., capacity approaching thresholds and linear distance growth.

AWGNC Thresholds vs. Distance Growth

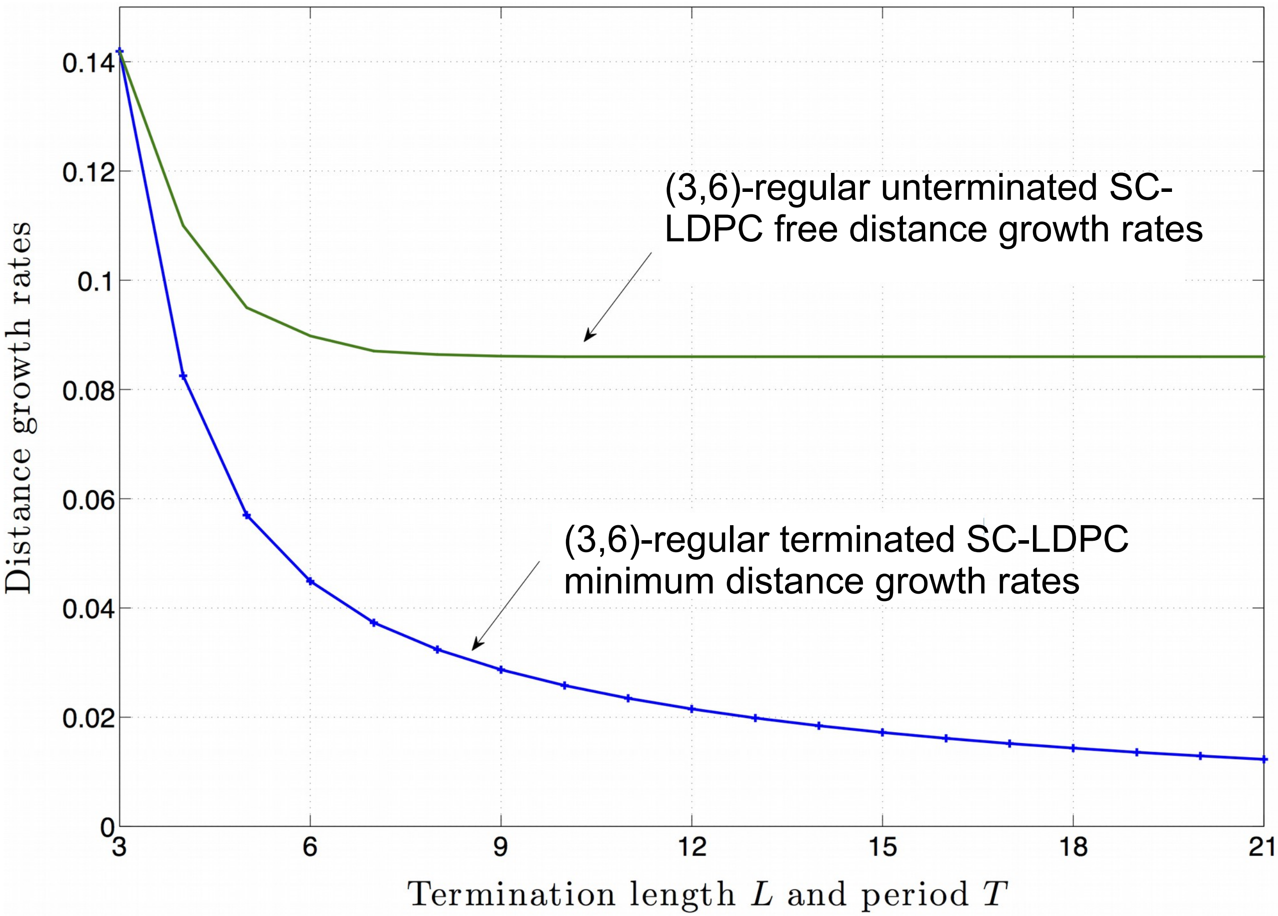
- Similar results are obtained for the AWGNC



[MLC10] D. G. M. Mitchell, M. Lentmaier and D. J. Costello, Jr., “AWGN Channel Analysis of Terminated LDPC Convolutional Codes”, *Proc. Information Theory and Applications Workshop*, San Diego, Feb. 2011.

Distance Measures for SC-LDPC Codes

- As $L \to \infty$ the minimum distance growth rates of **terminated** SC-LDPC code ensembles tend to zero. However, the free distance growth rates of the **unterminated** ensembles remain constant.



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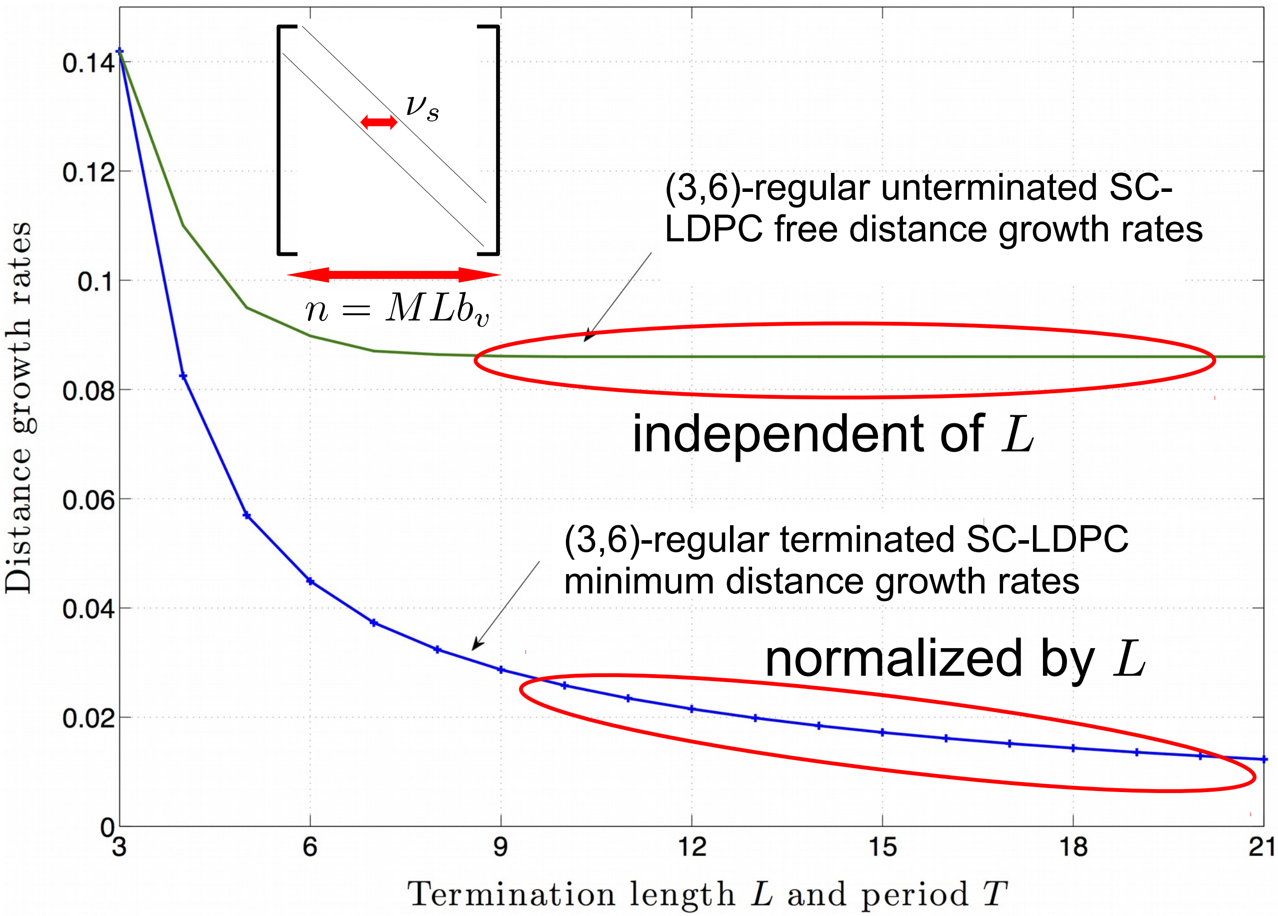

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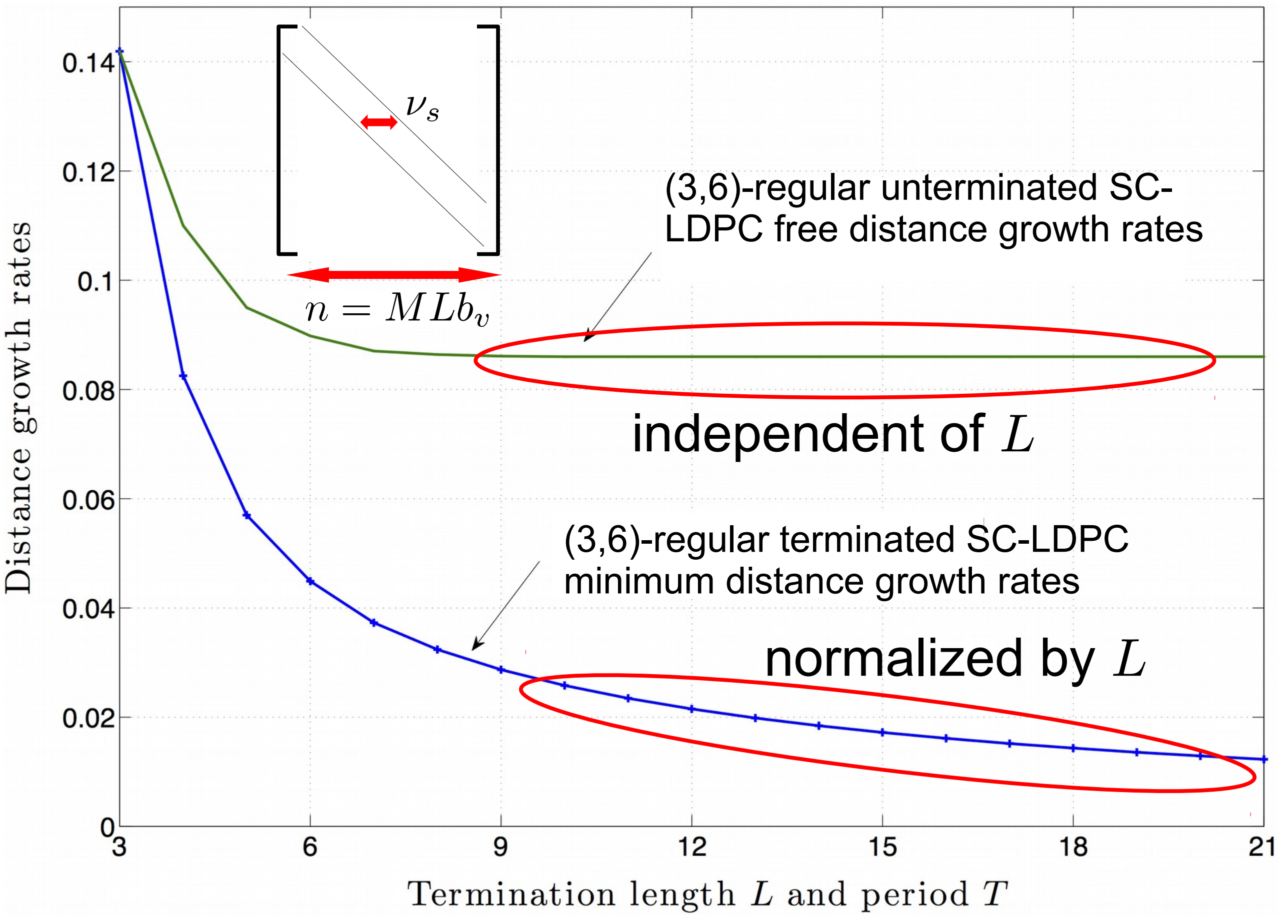


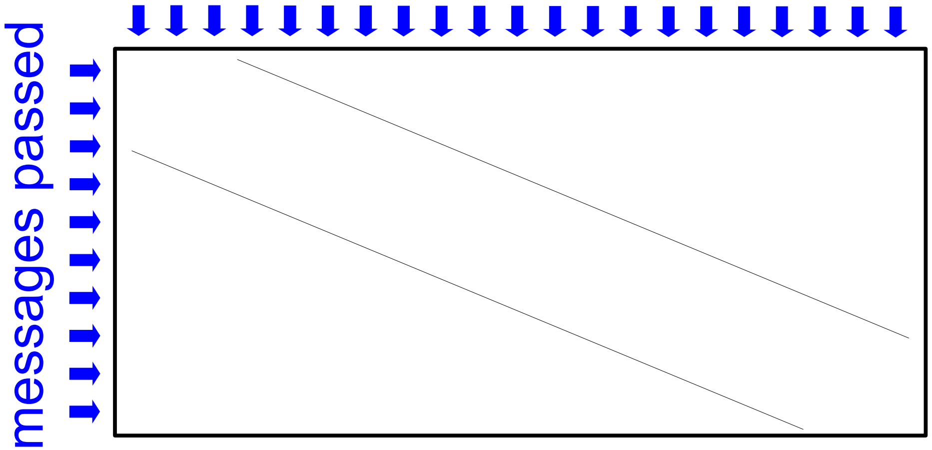
- For large L, the strength of unterminated ensembles **scales with the constraint length** $\nu_s = M(m_s+1)b_v$ and is **independent** of L.
- An appropriate distance measure for 'convolutional-like' terminated ensembles should be independent of L.

Outline

- **LDPC Block Codes**
  - ➡ Parity-check matrix and Tanner graph representations, minimum distance bounds, iterative decoding thresholds, protograph-based constructions
- **Spatially Coupled LDPC Codes**
  - ➡ Protograph representation, edge-spreading construction, termination
  - ➡ Iterative decoding thresholds, threshold saturation, minimum distance
- **Practical Considerations**
  - ➡ Window decoding; performance, latency, and complexity comparisons to LDPC block codes; rate-compatibility; implementation aspects

Block Decoding of SC-LDPC Codes

- SC-LDPC codes can be decoded with standard iterative decoding schedules.



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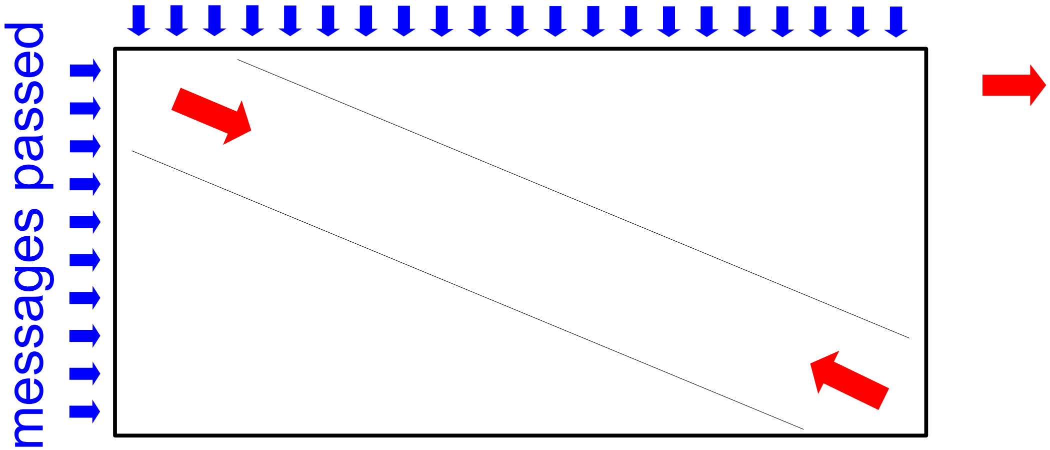

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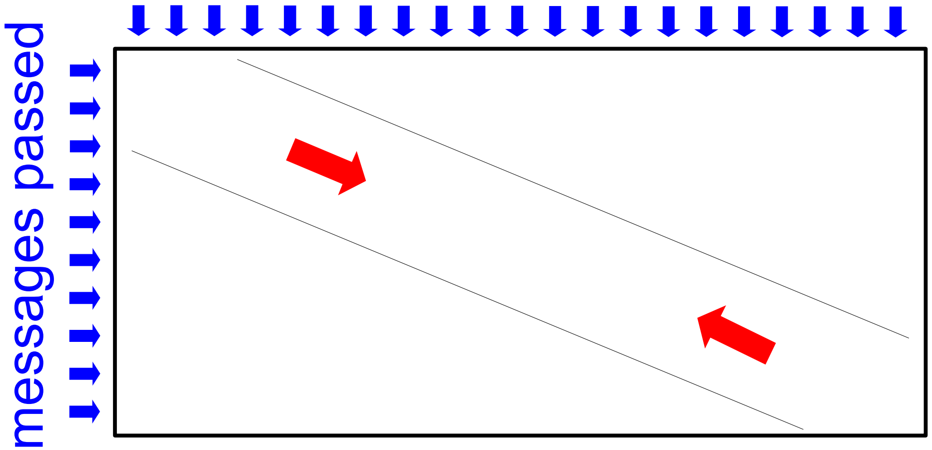

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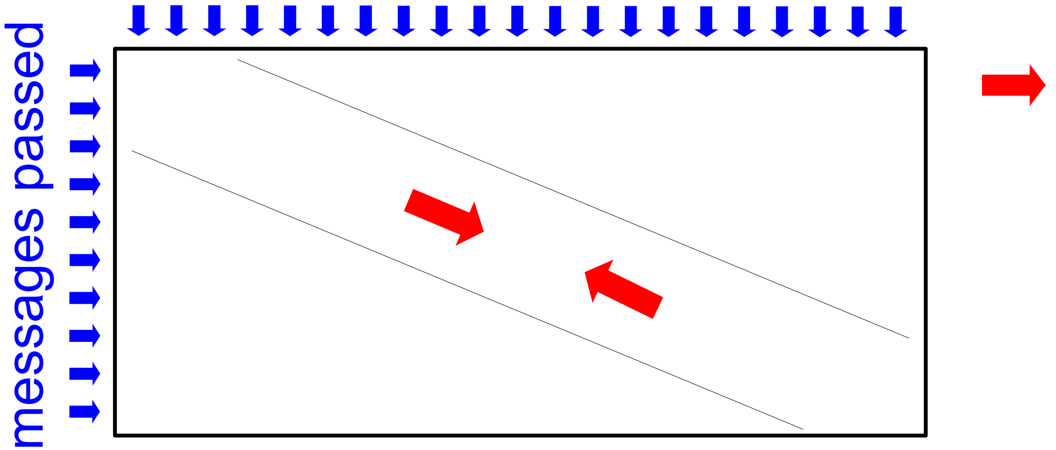

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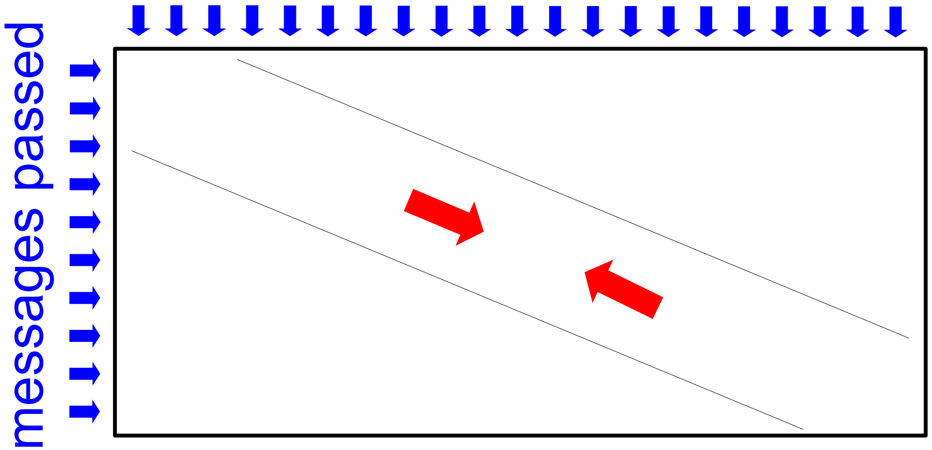
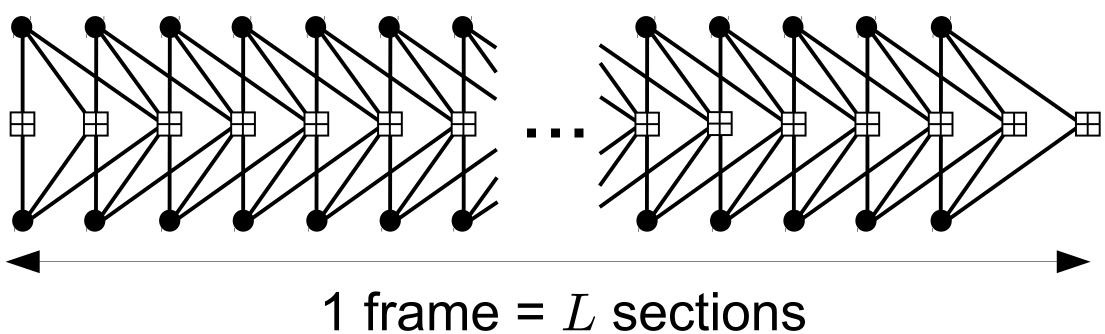


➡ Reliable messages from the ends propagate through the graph toward the center as iterations proceed.

- The **frame error rate (FER)** of a terminated graph can be analyzed



➡ The **FER depends on L** (FER $\underset{L\to\infty}{\longrightarrow} 1$)

Block Decoding Performance

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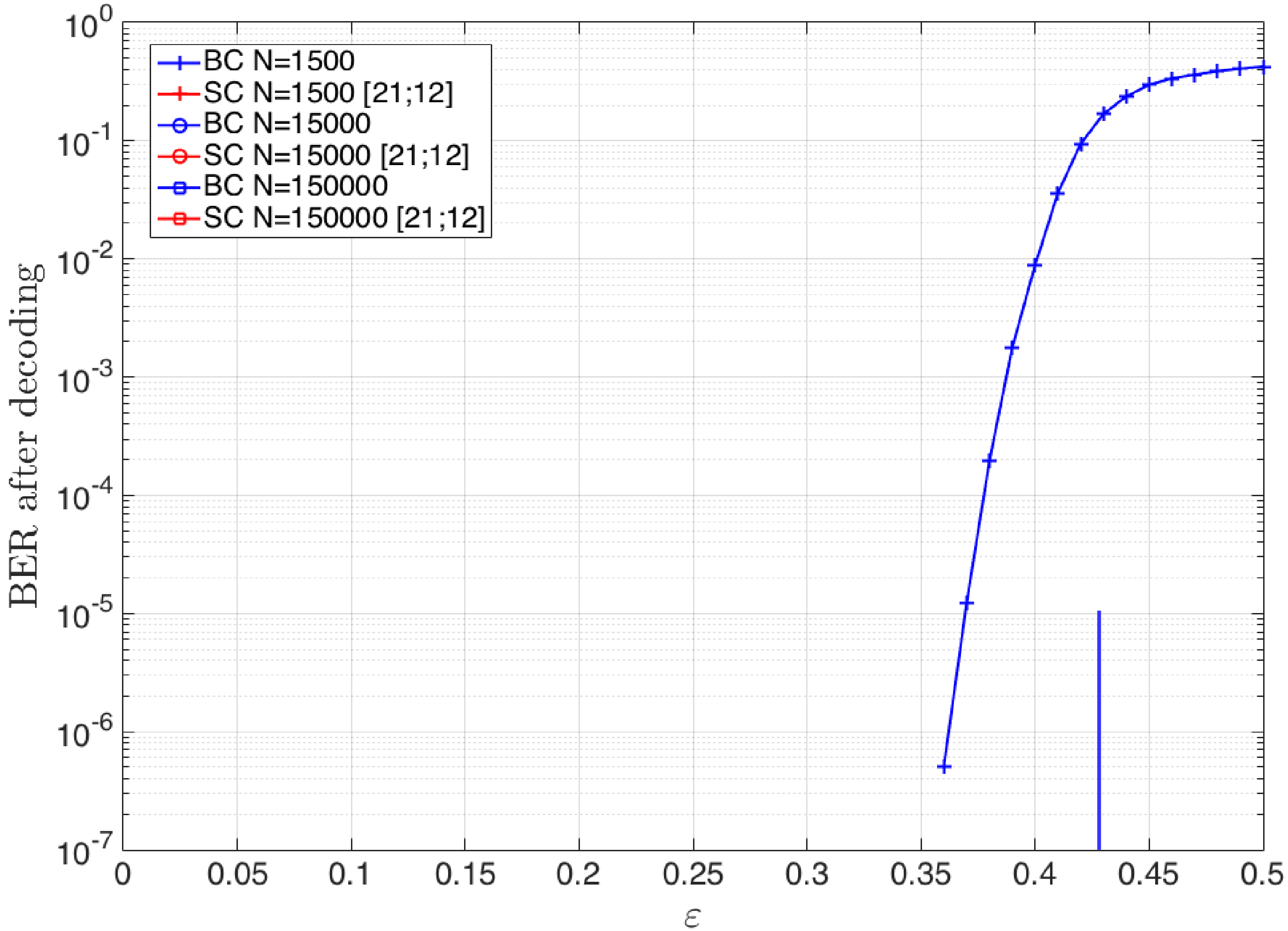
- Consider LDPC-BCs and SC-LDPC codes with increasing frame length N



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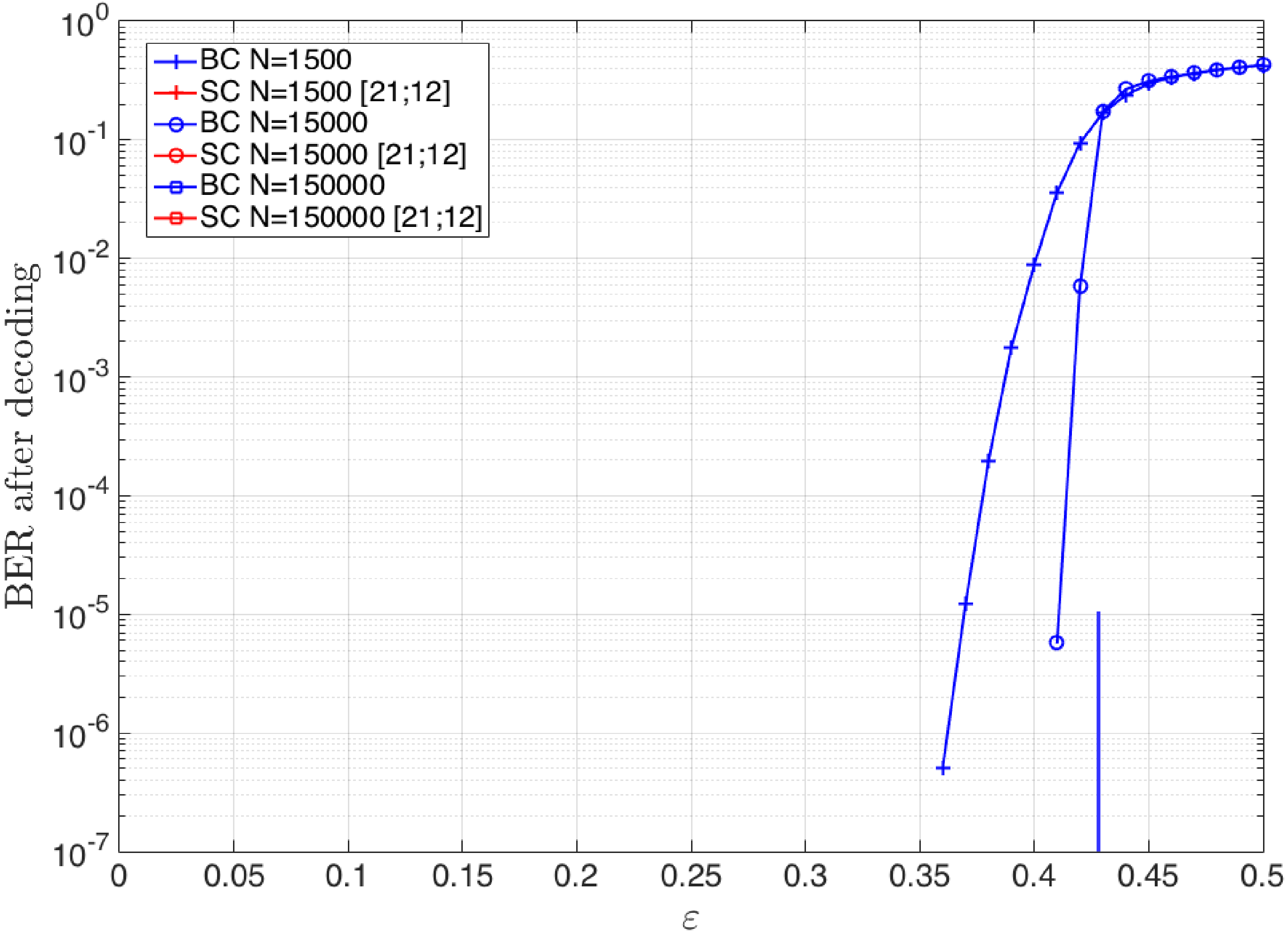

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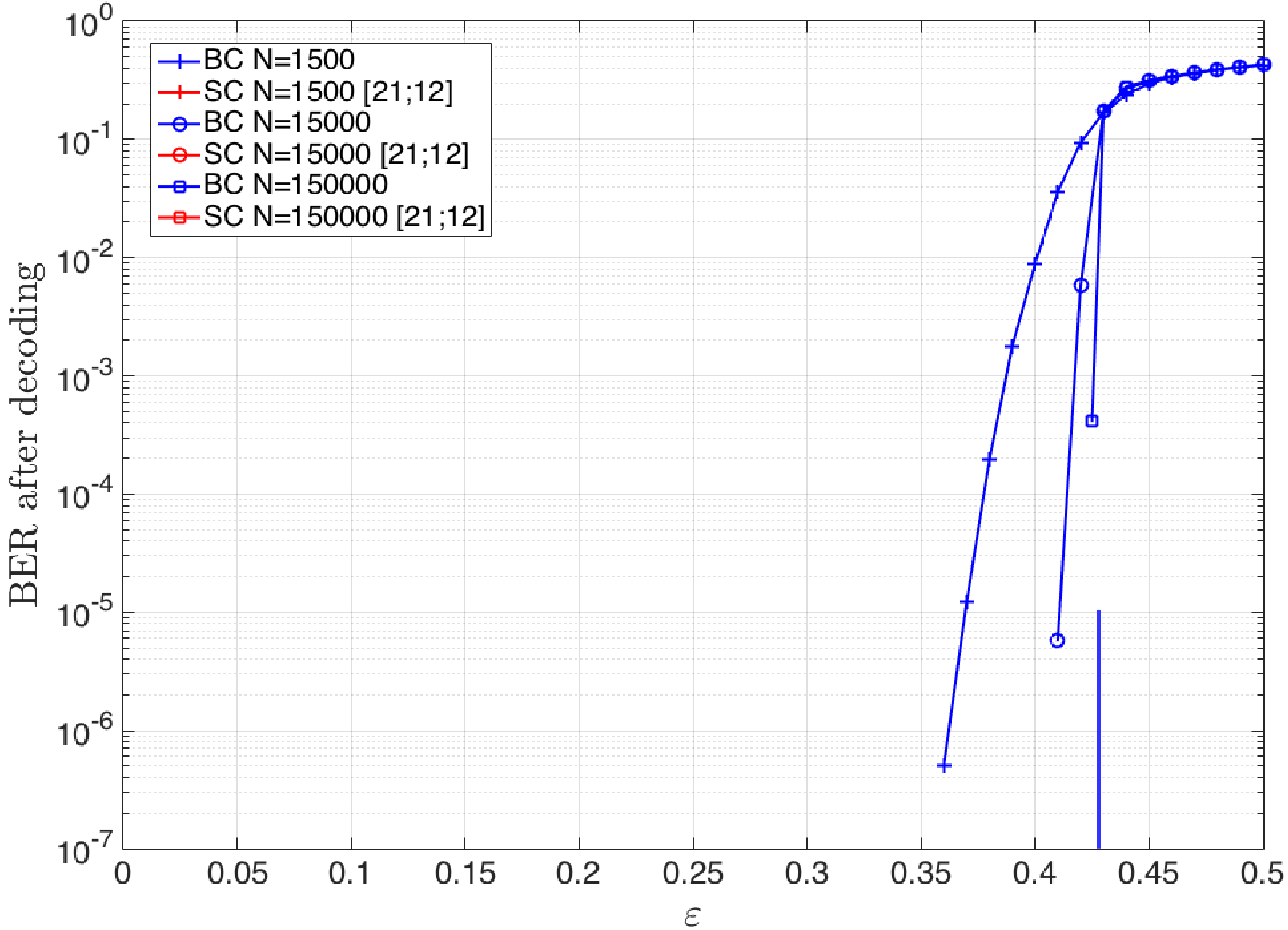

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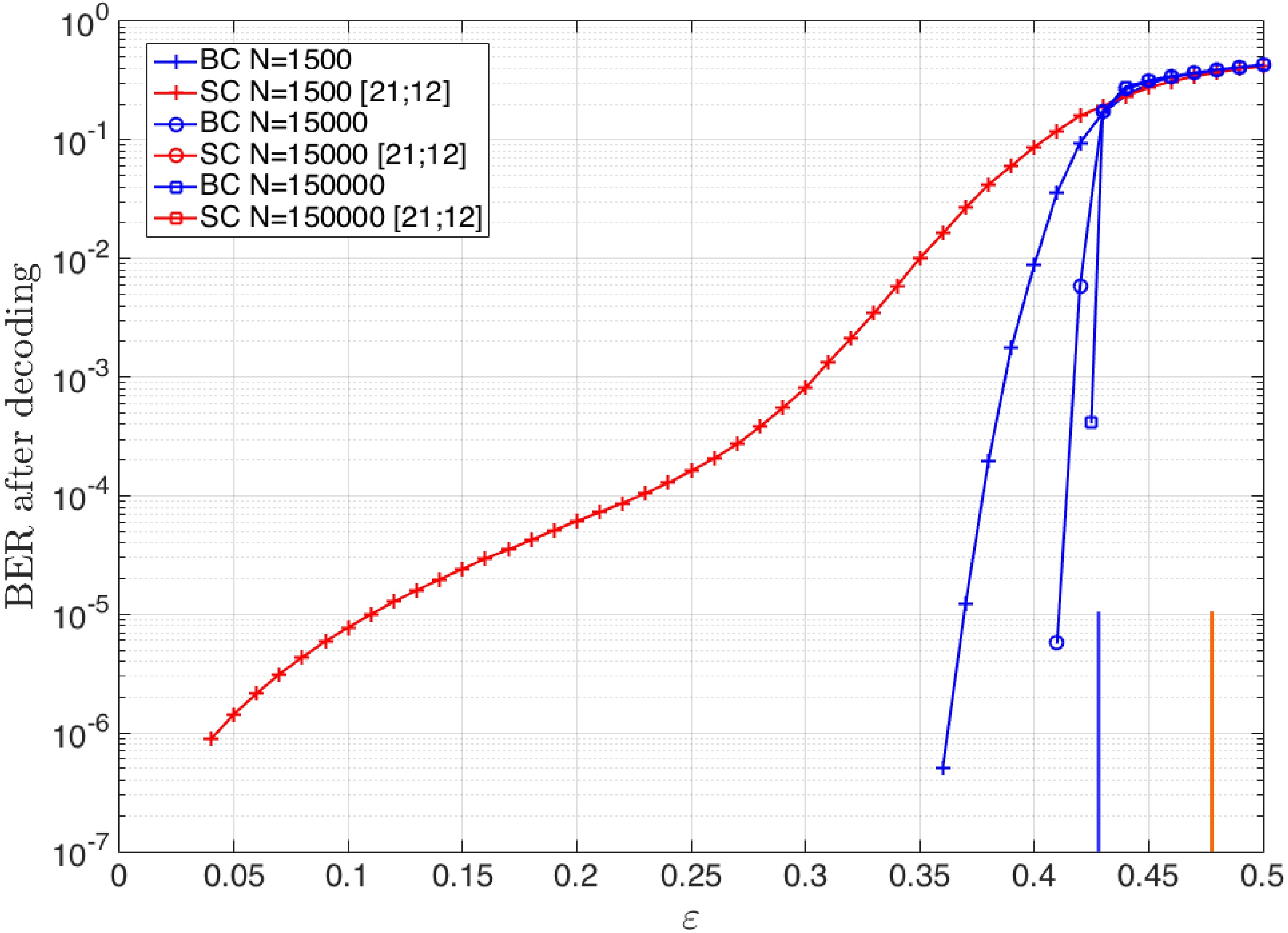

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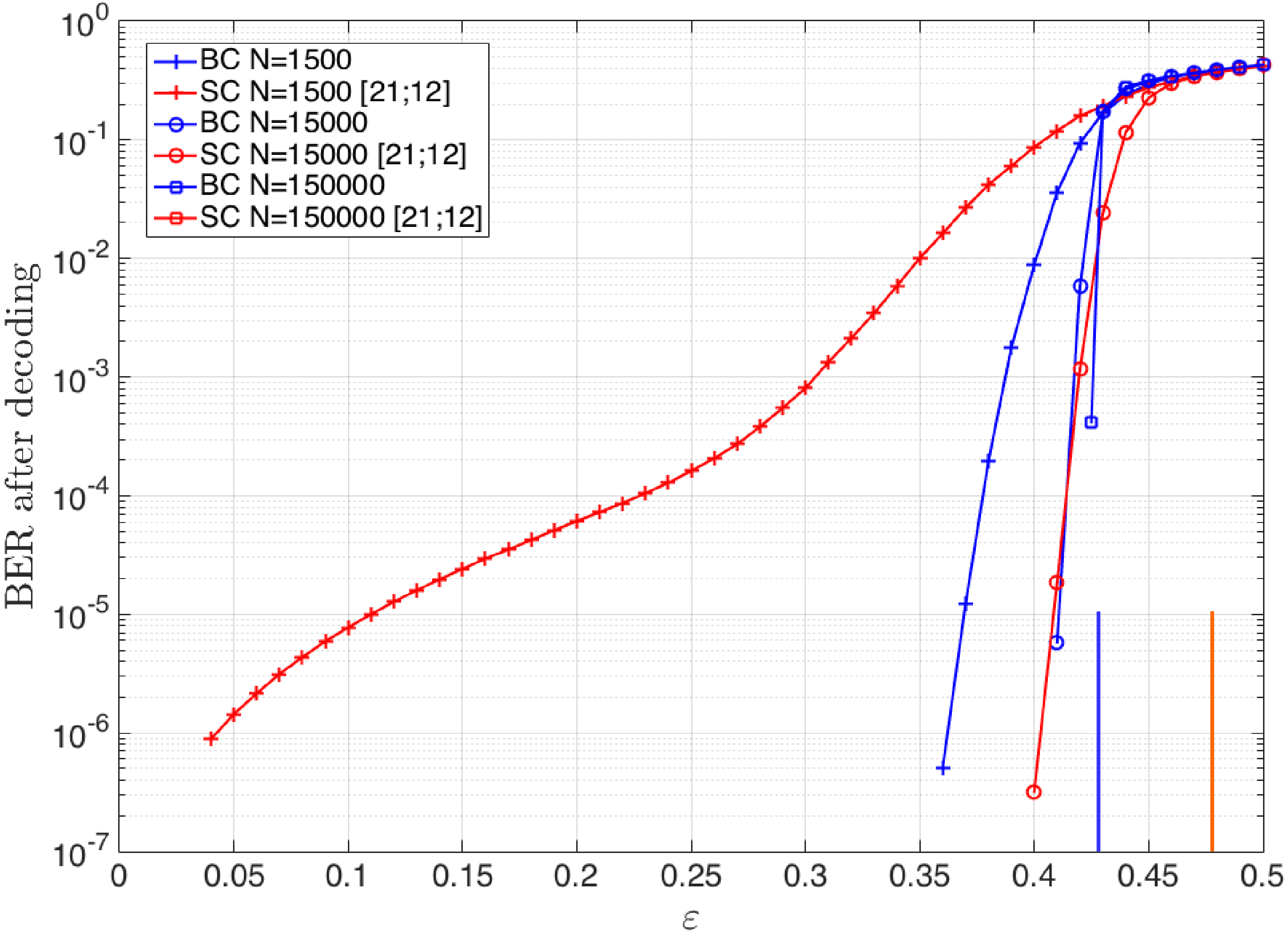

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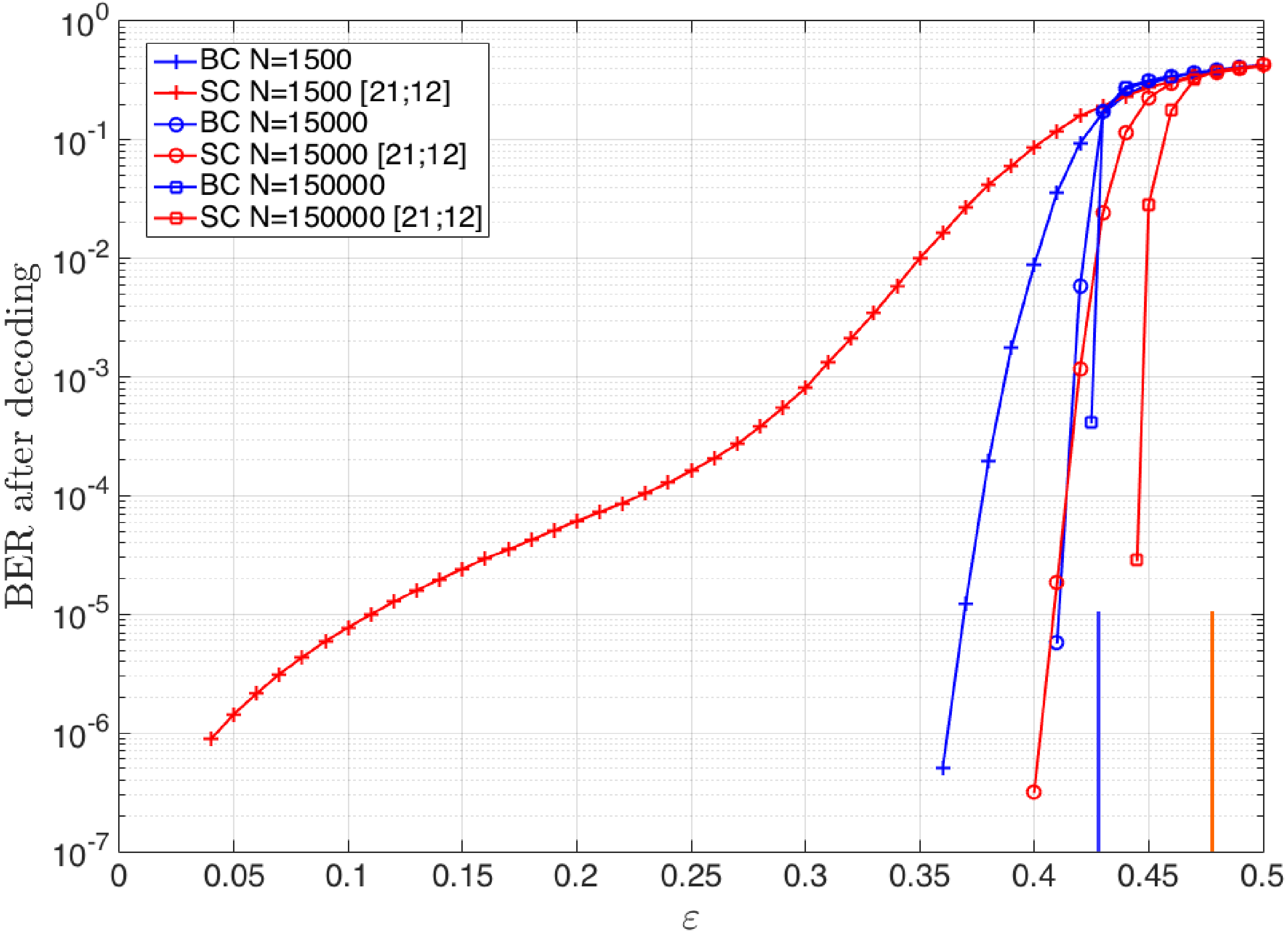

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Window Decoding of SC-LDPC Codes

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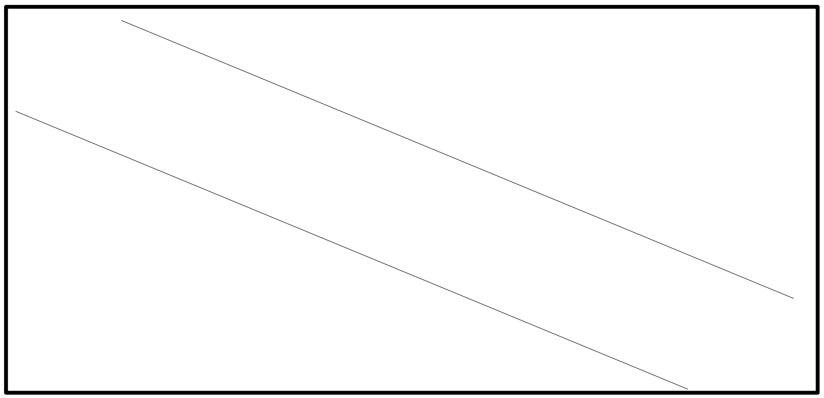

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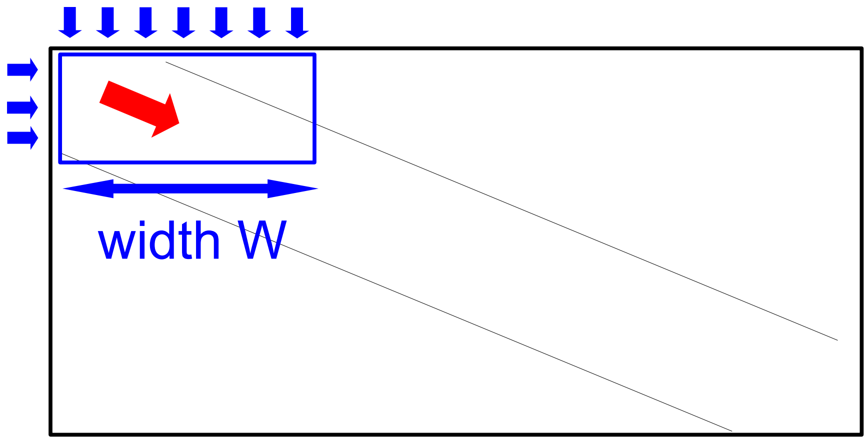

➡ **Sliding window decoding** (WD) updates nodes only within a localized window and then the window shifts across the graph [Lentmaier et al '10, Iyengar et al '12].

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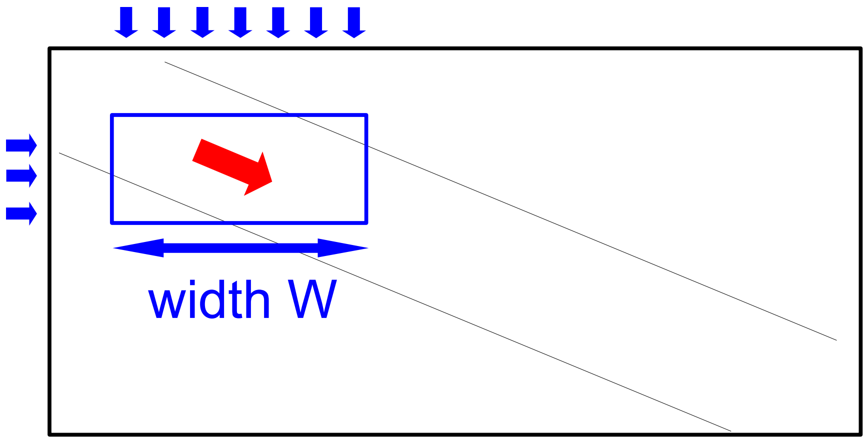

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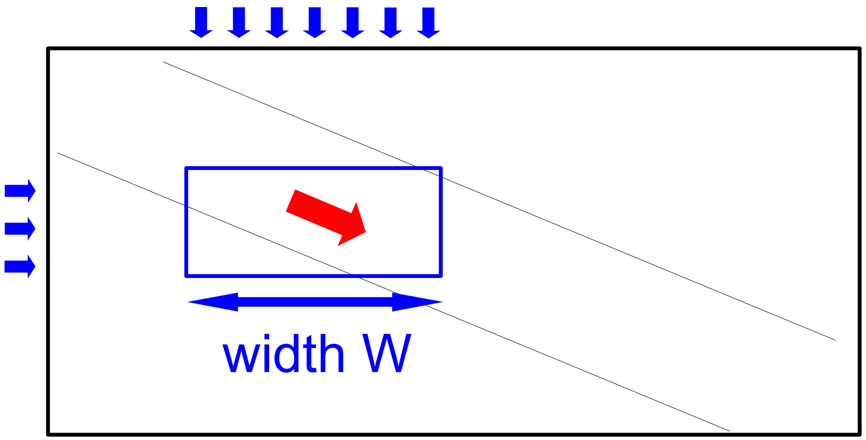

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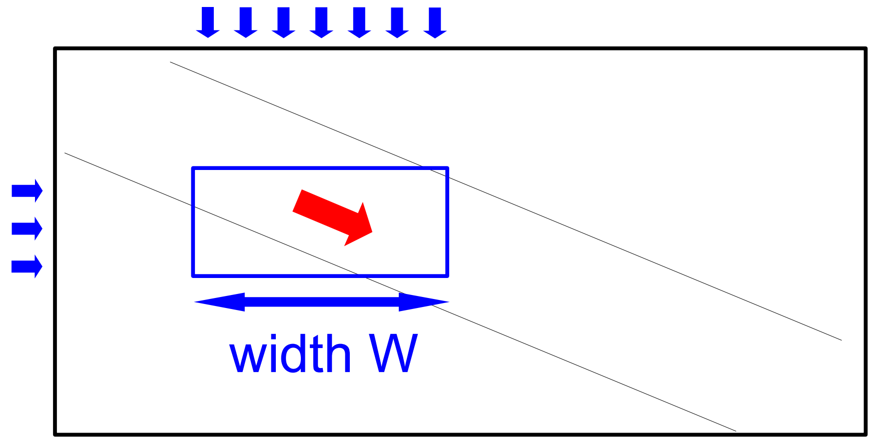

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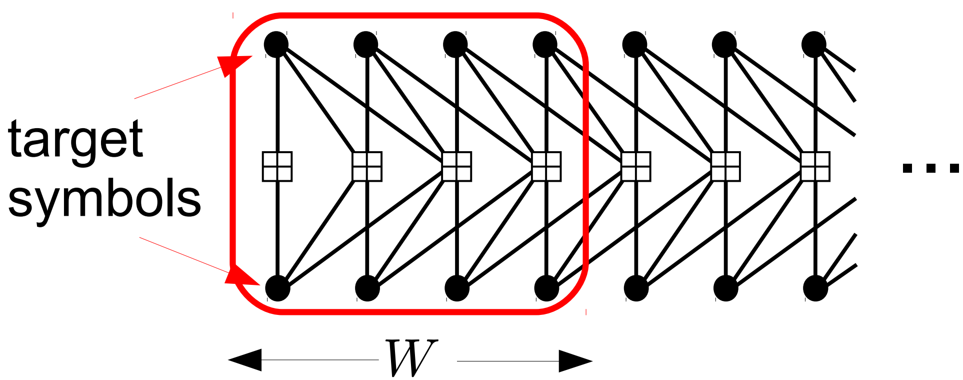

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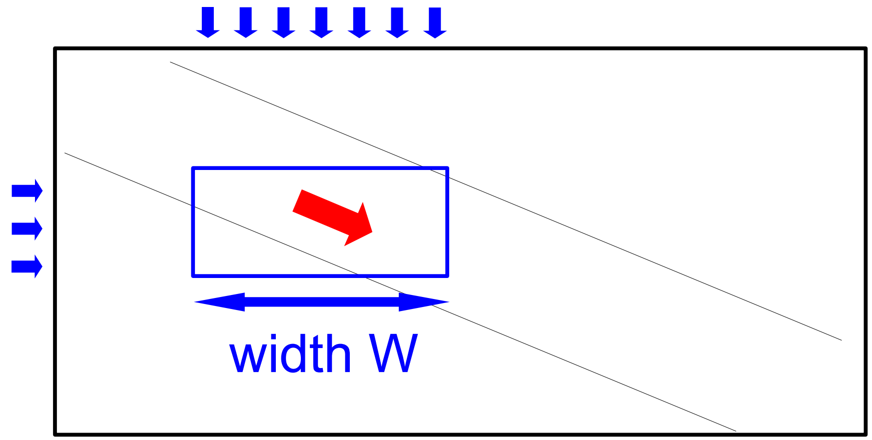

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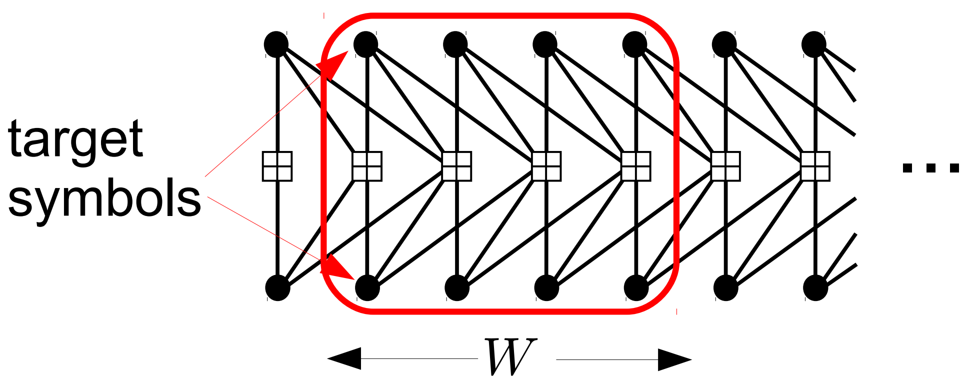

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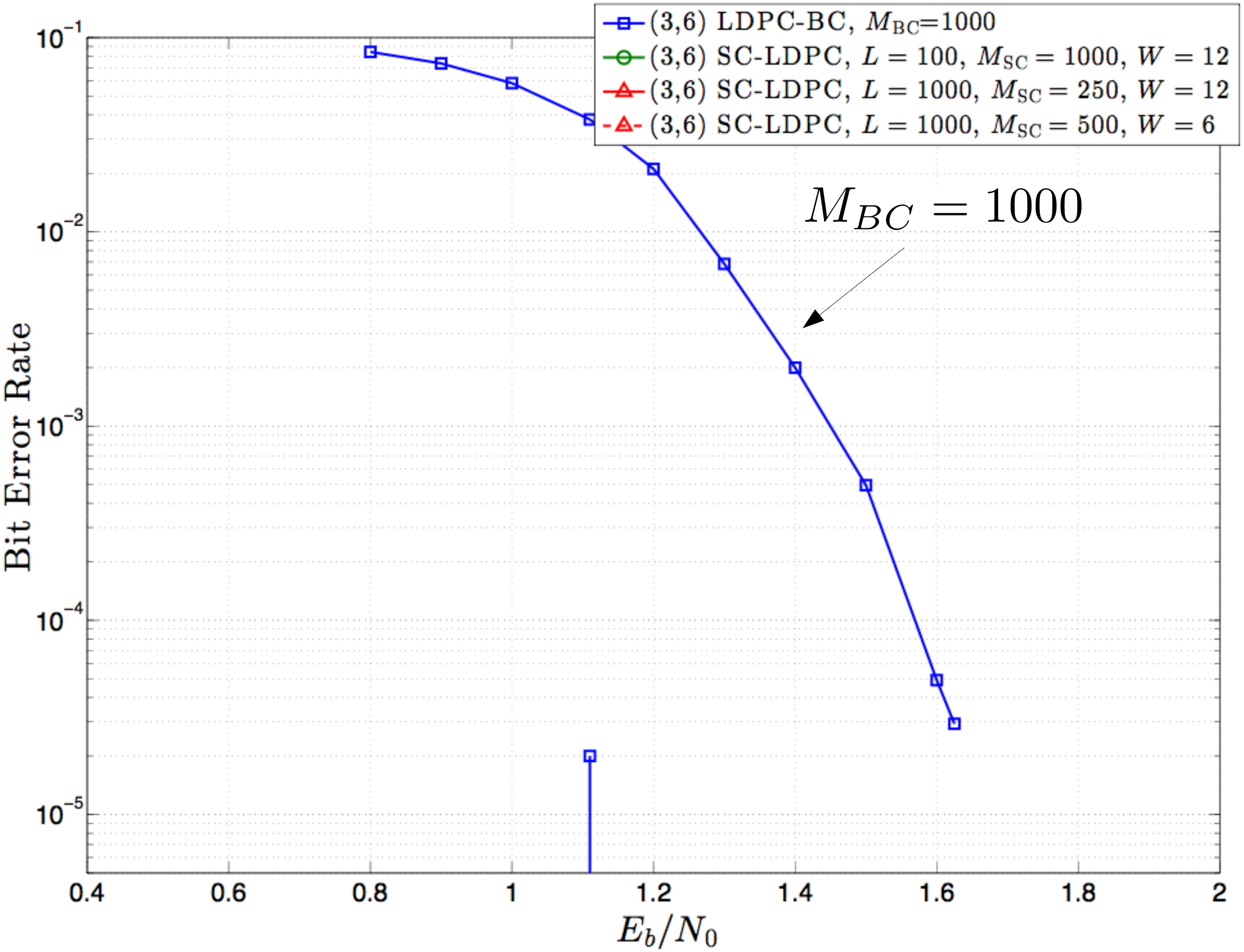
➡ The window then shifts to the right

Window Decoding Performance



Latencies:
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[LPF11] M. Lentmaier, M. M. Prenda, and G. Fettweis, "Efficient Message Passing Scheduling for Terminated LDPC Convolutional Codes", *Proc. IEEE ISIT*, St. Petersburg, Russia, July 2011.

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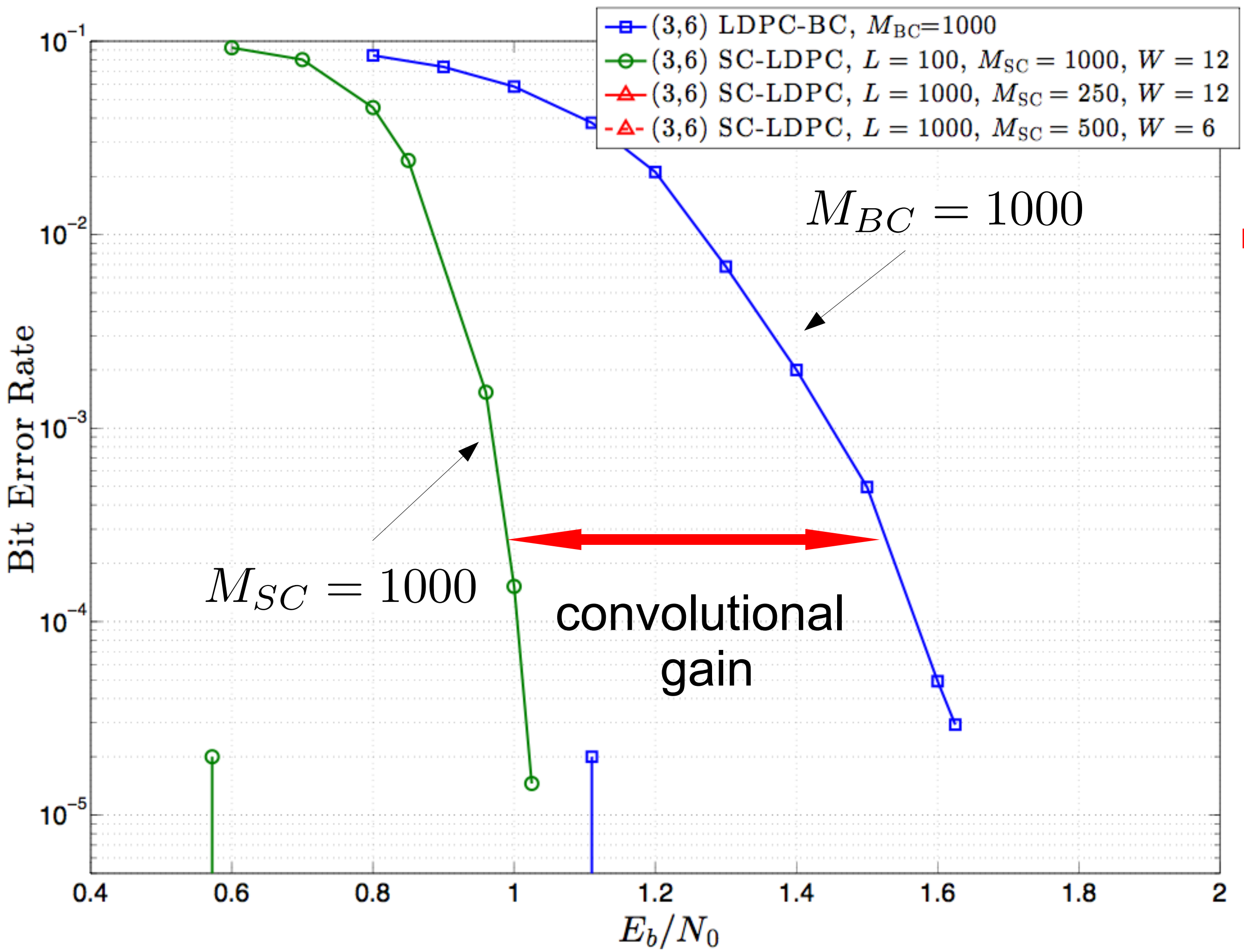

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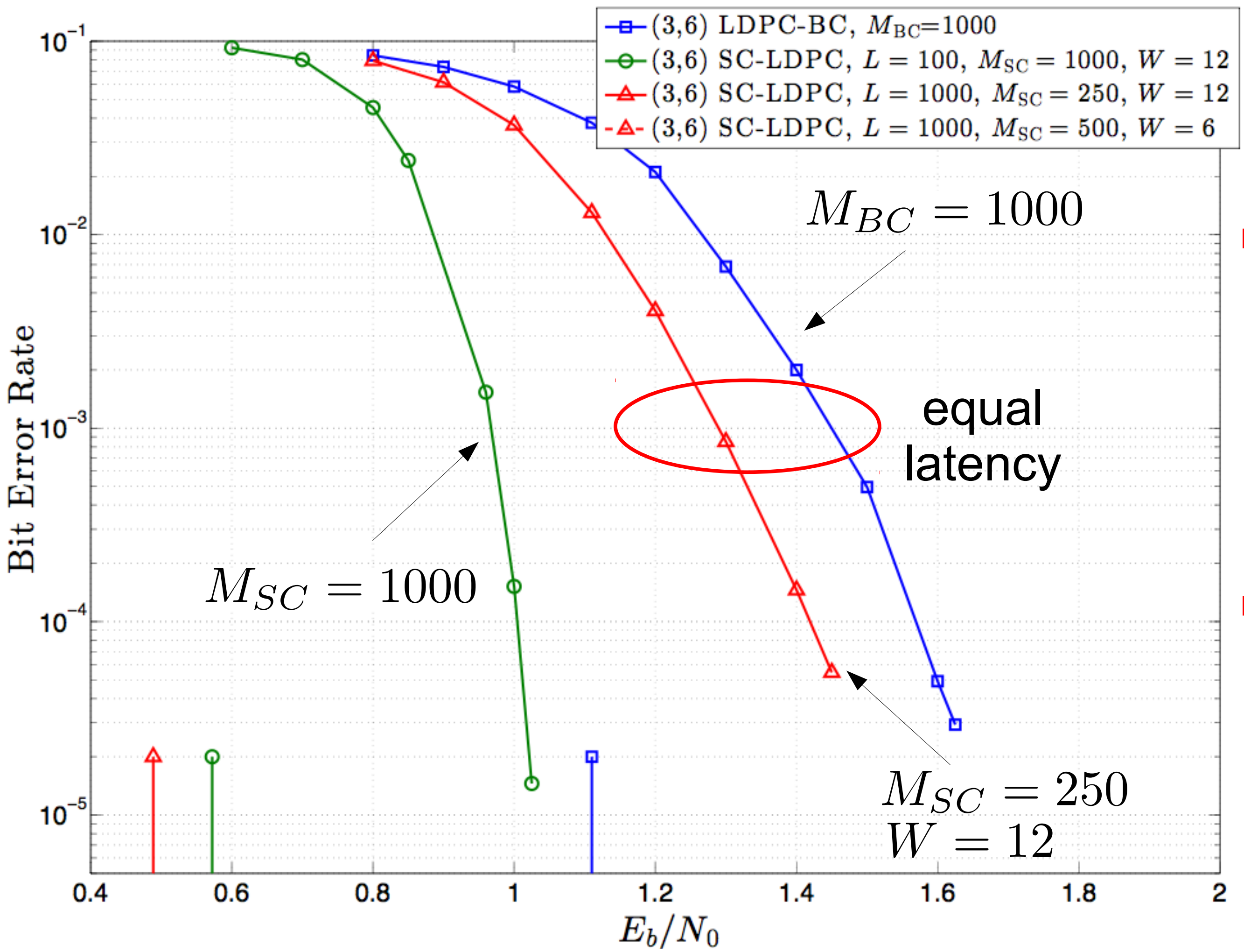

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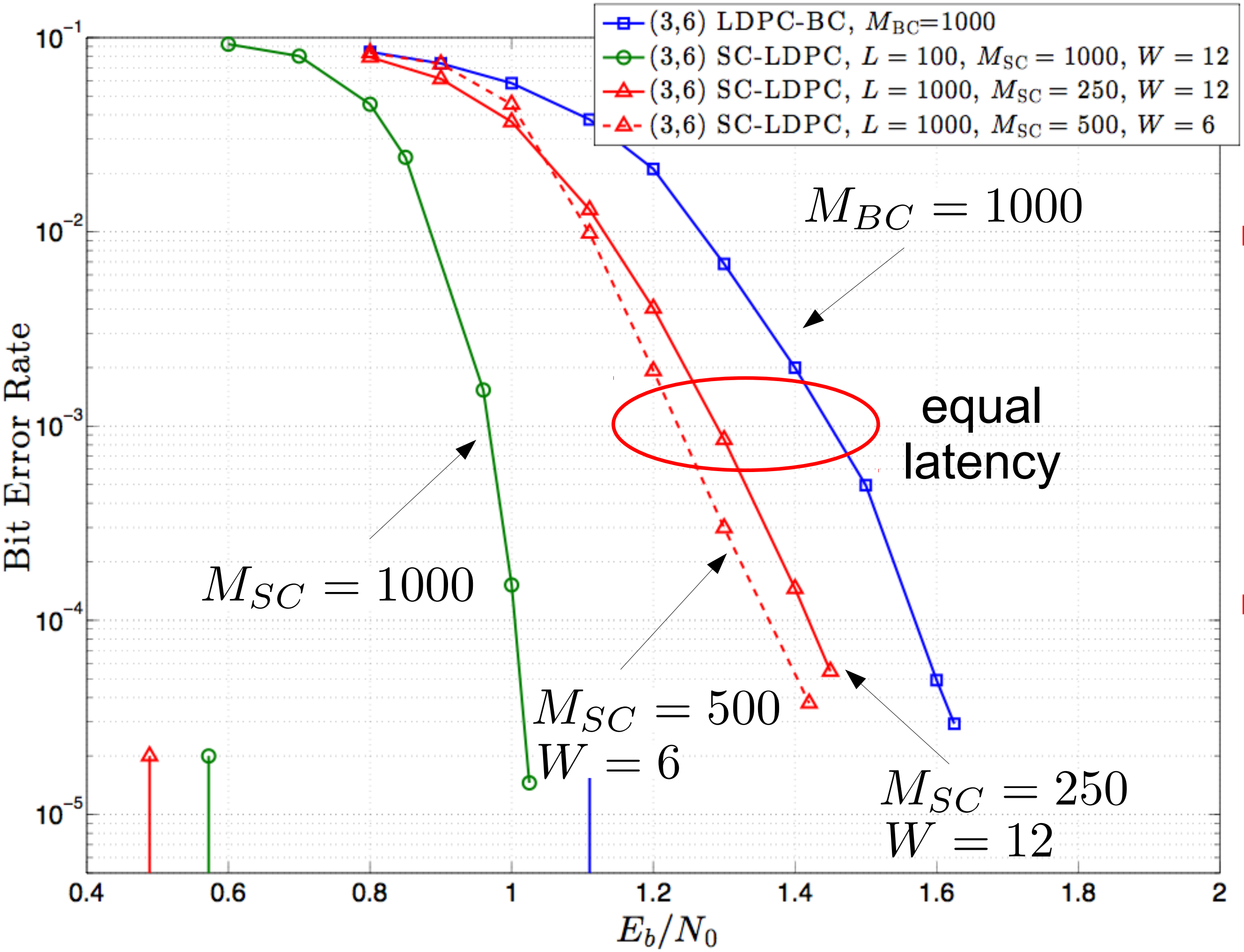

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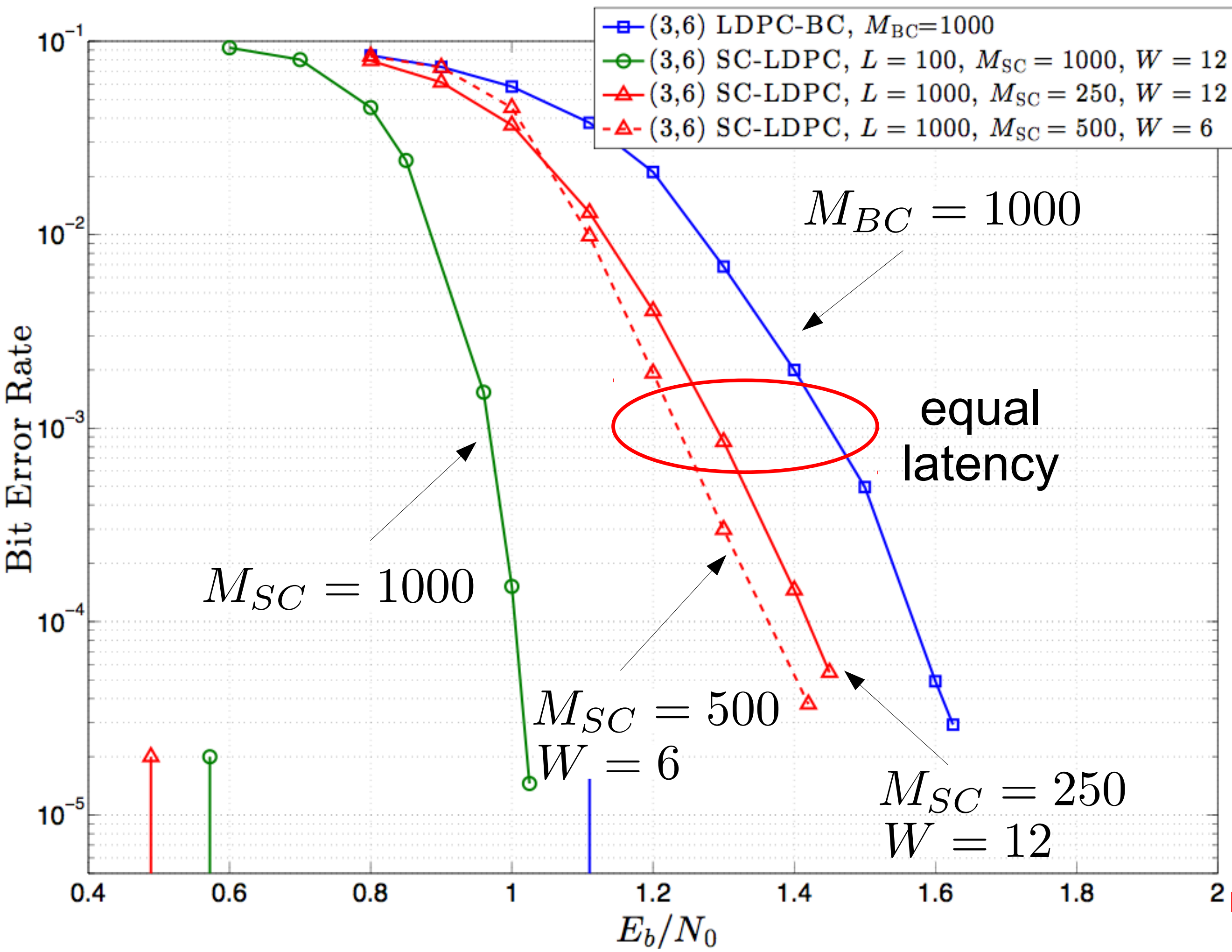

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- Trade-off in M vs W

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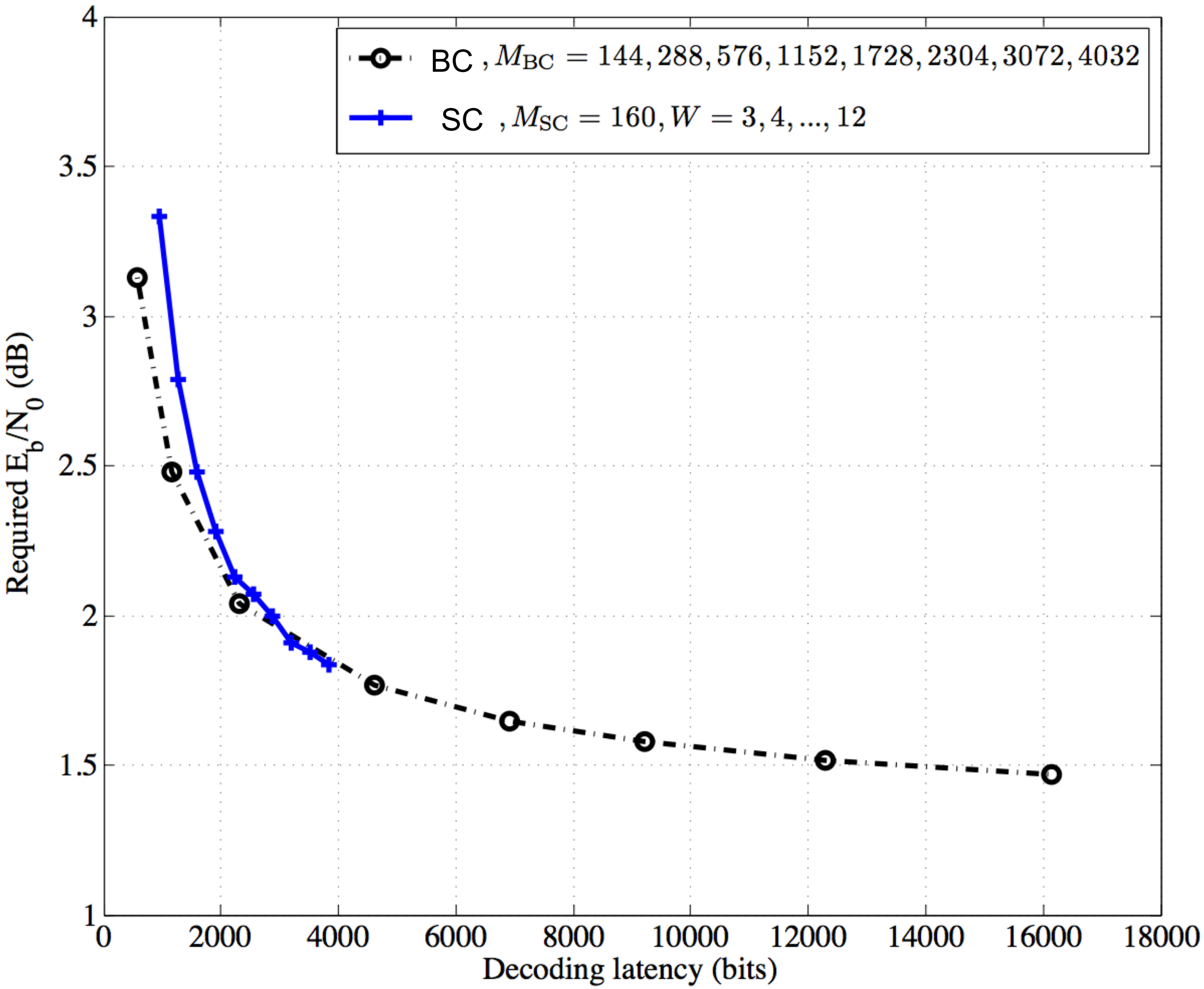
Equal Latency Comparison for (3,6)-Regular LDPC Codes

- Required E_b/N_0 to achieve a BER of 10^{-5} as a function of latency:



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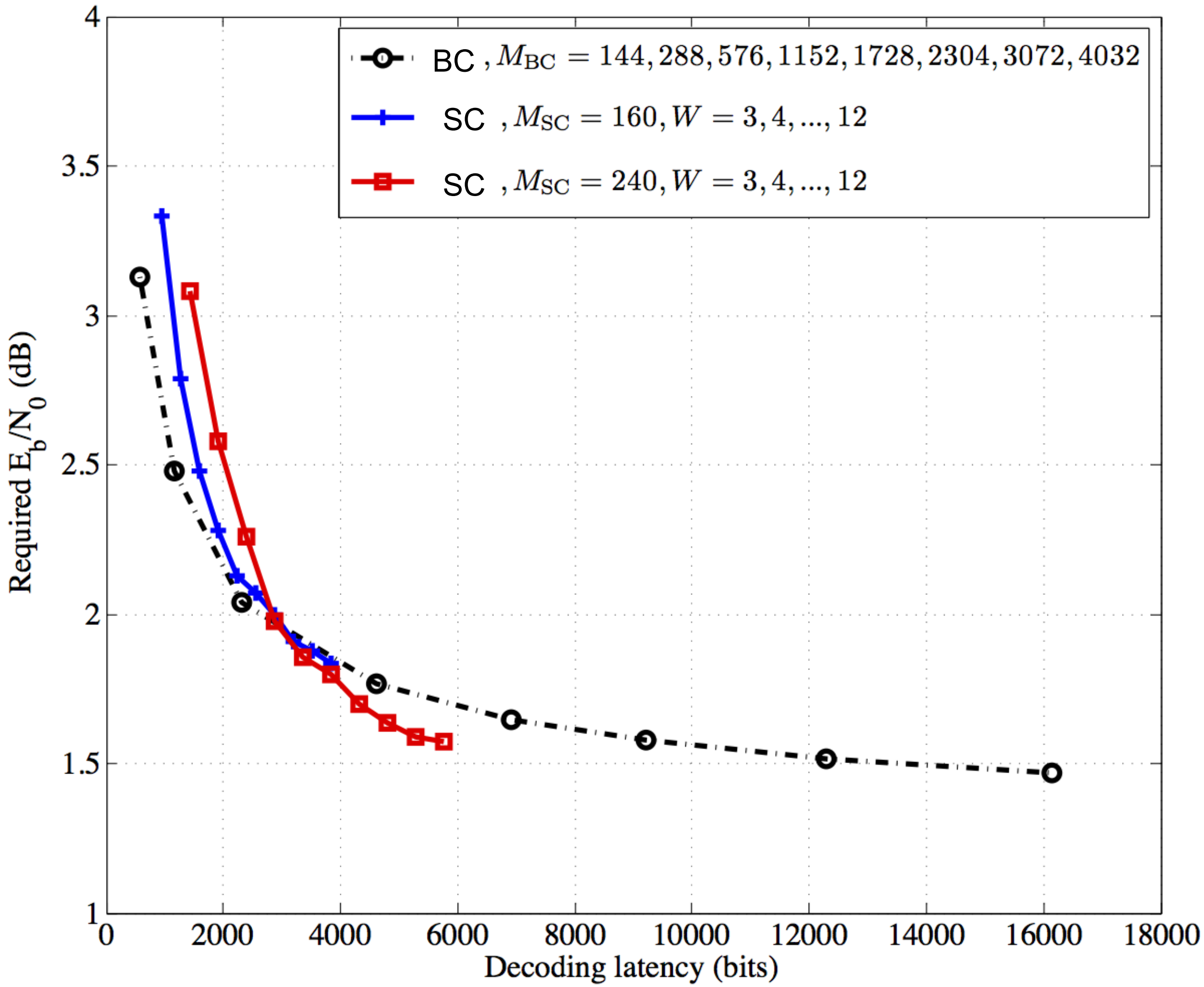

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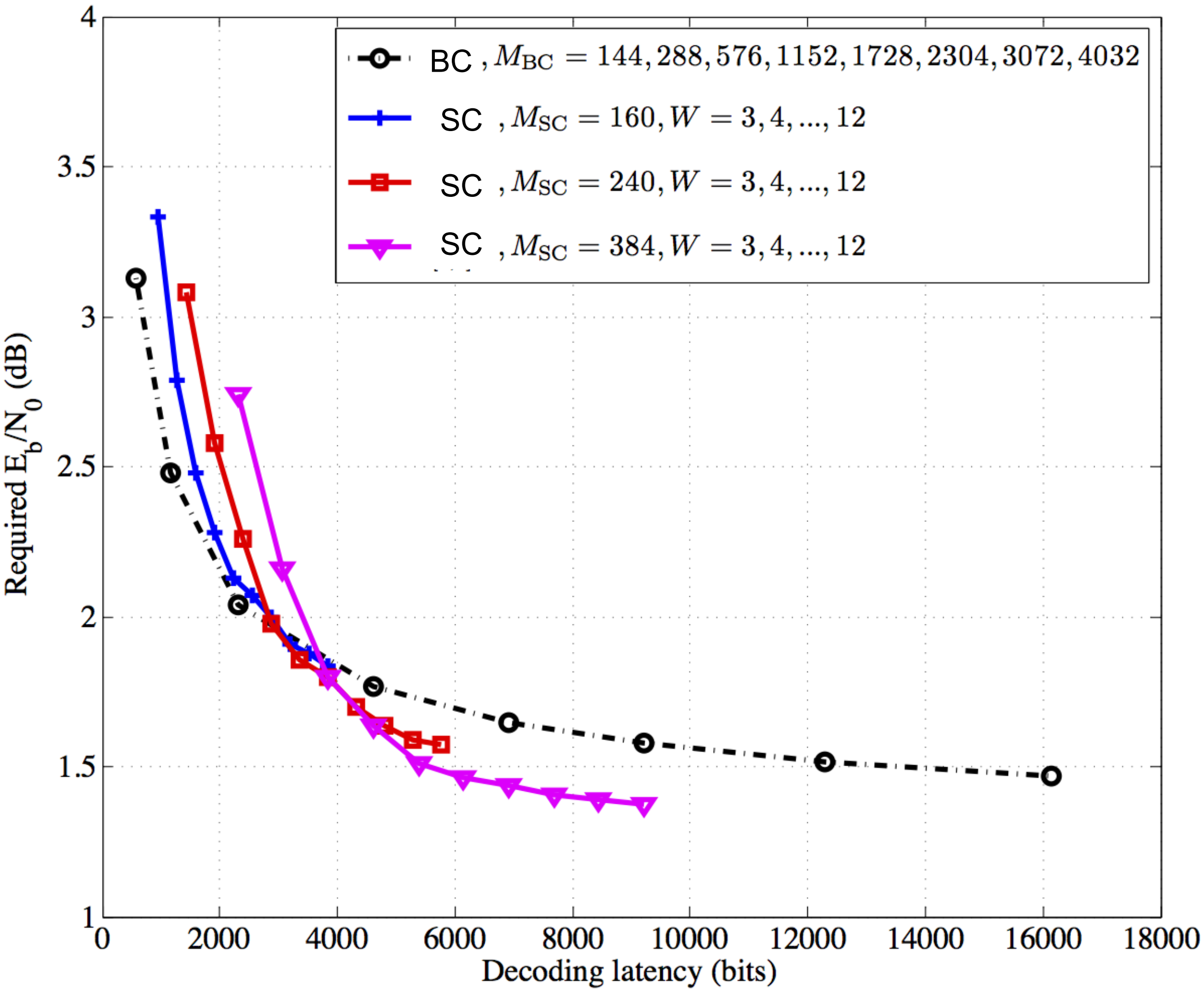

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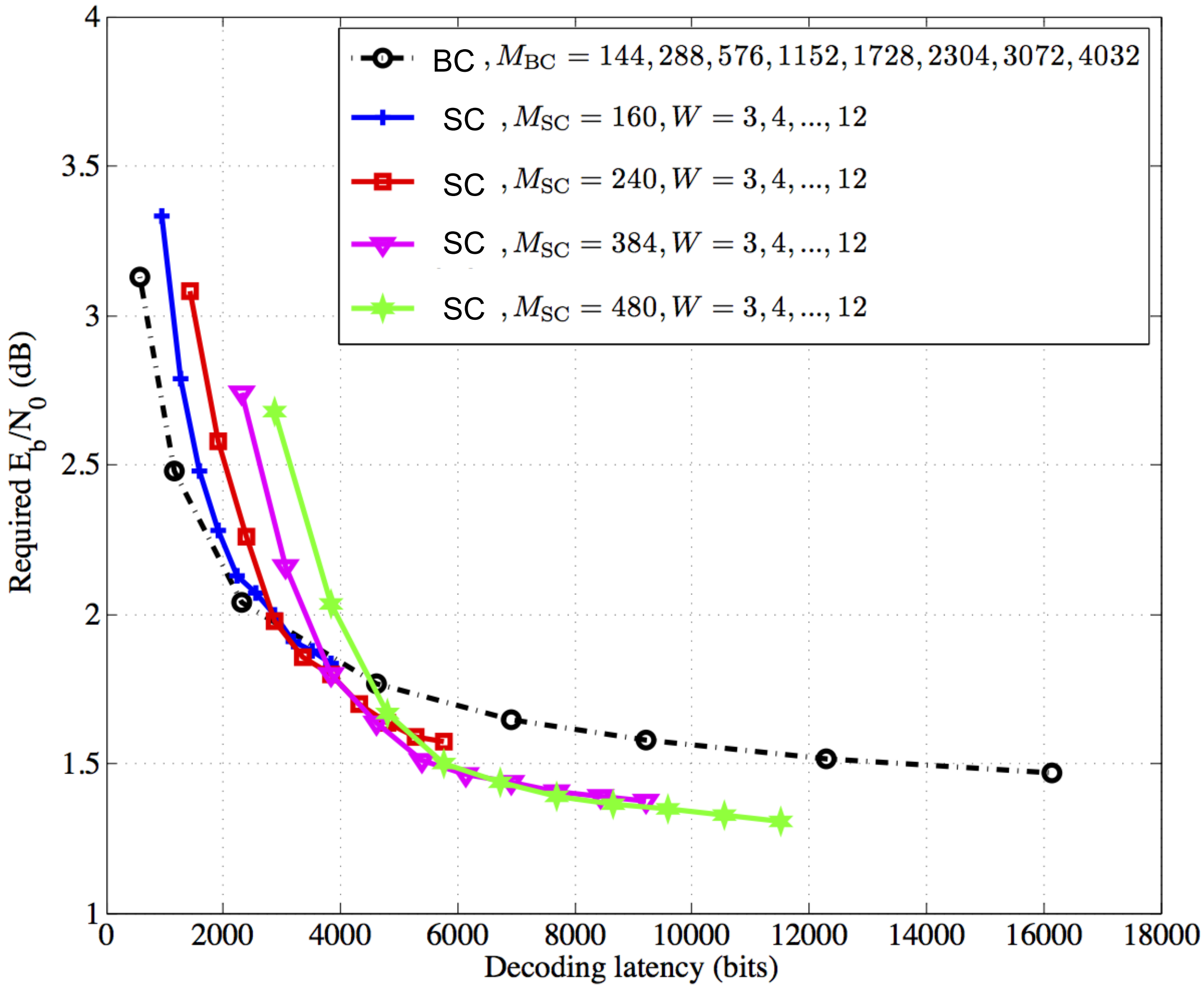

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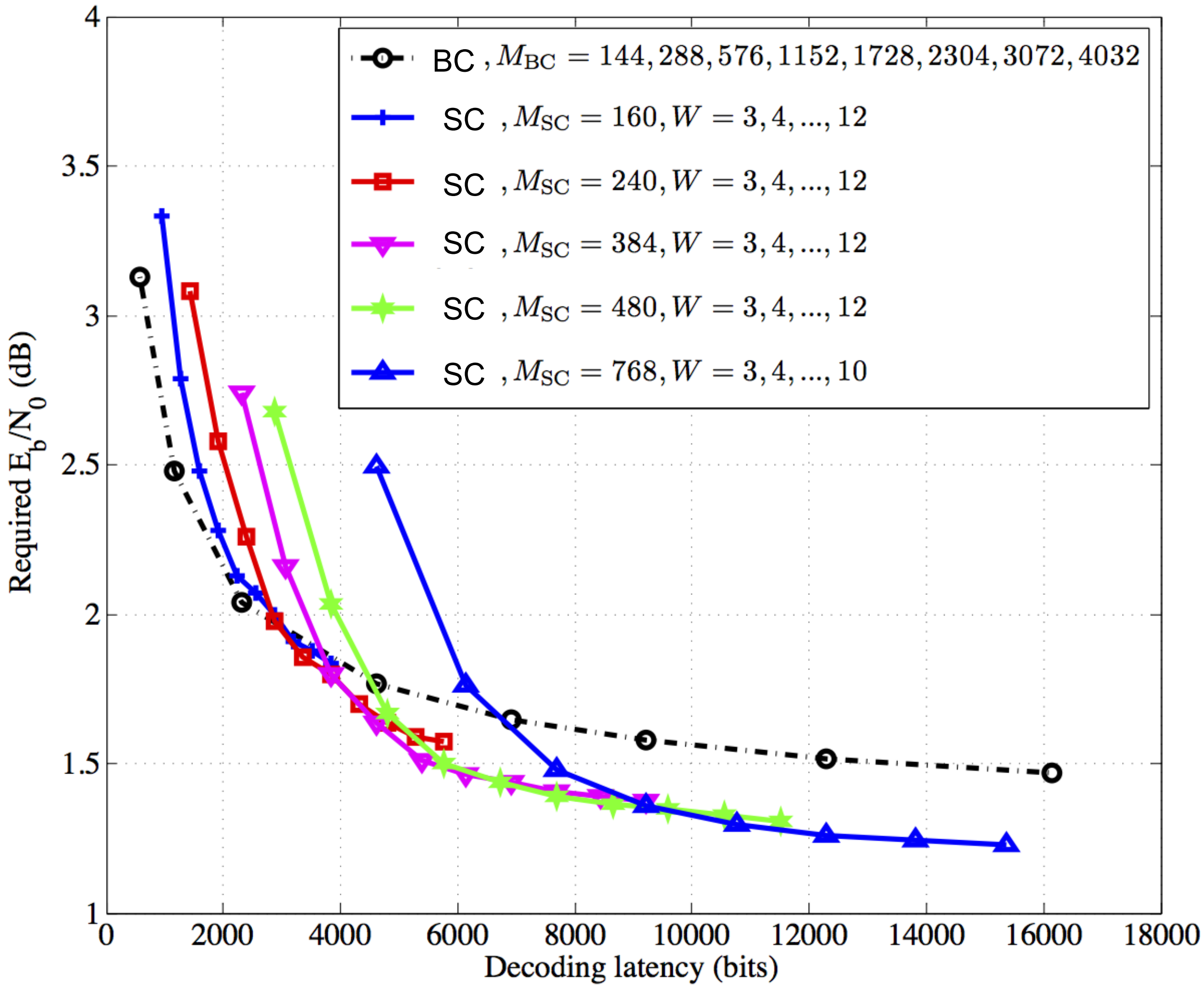

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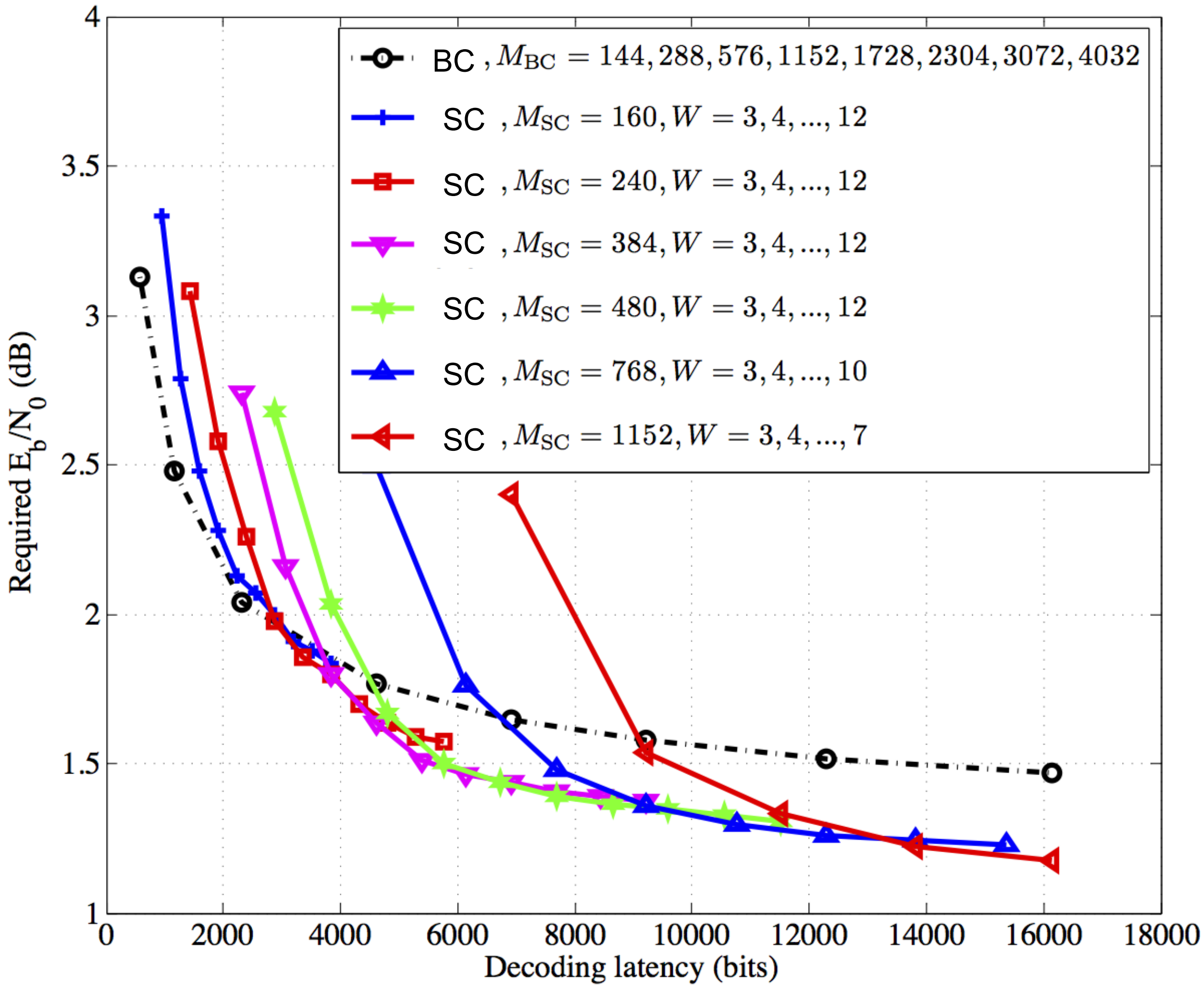

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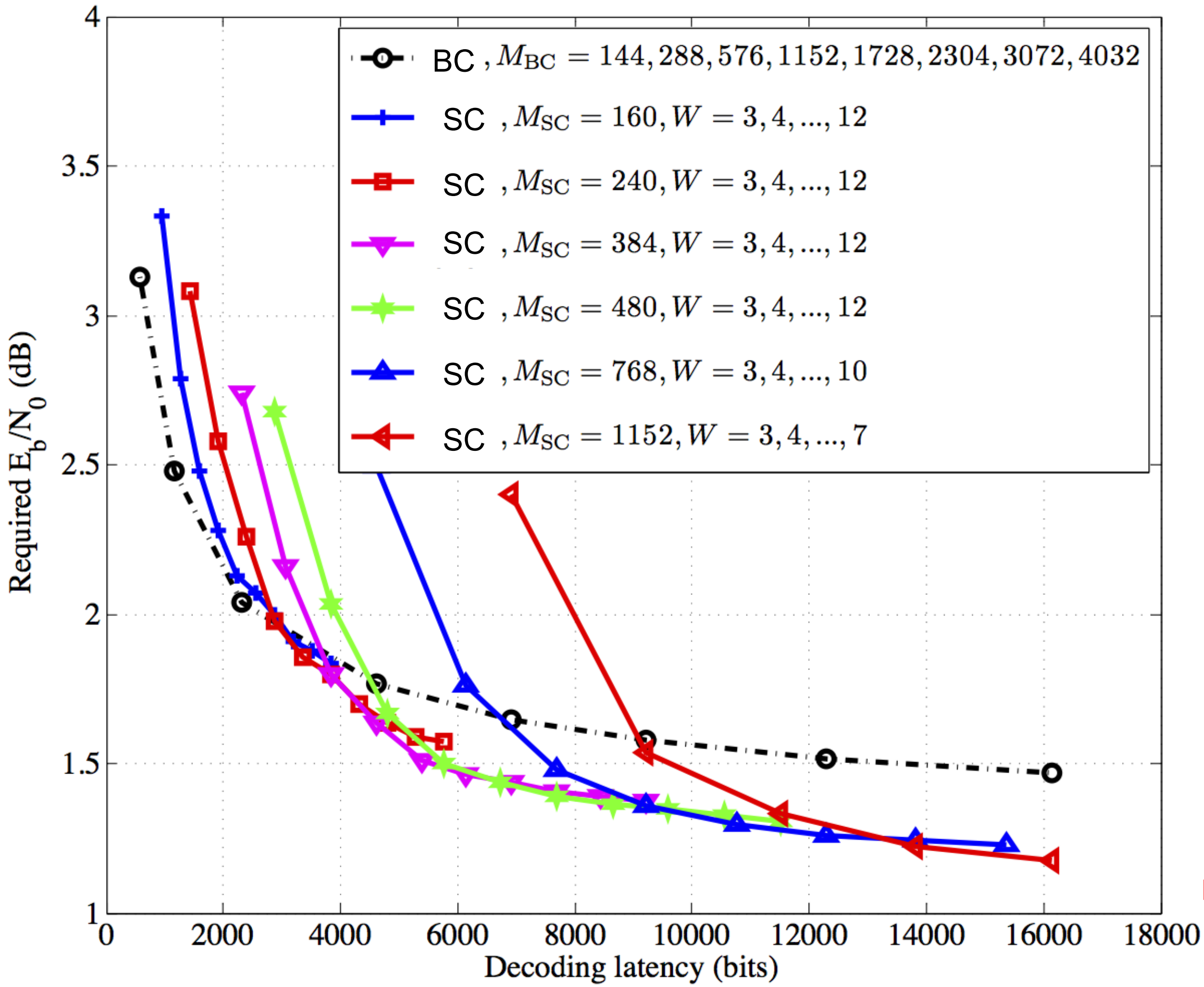

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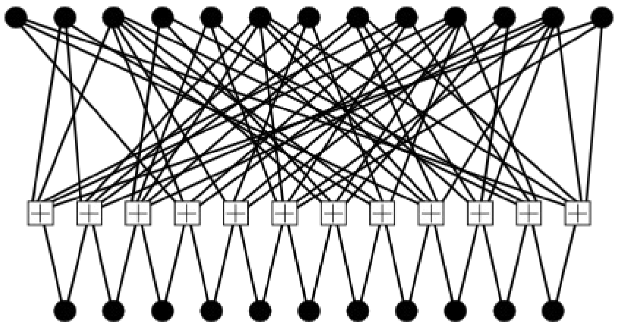
- When choosing parameters:
  - ➡ large M_{SC} improves code performance.
  - ➡ large W improves decoder performance.

Regular SC-LDPC Codes vs. Irregular LDPC-BCs

- Consider a comparison of a (3,6)-regular SC-LDPC code vs. an irregular-repeat-accumulate (IRA) LDPC-BC with optimized protograph taken from the WiMAX standard



Ex: $M = 250$
$n = 24M = 6000$

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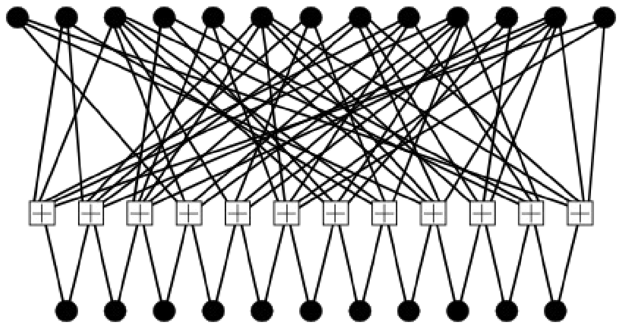

Ex: $M = 250$
$n = 24M = 6000$

- The IRA LDPC-BC ensemble has rate R=0.5, BEC threshold $\epsilon^* \approx 0.4489$, and AWGNC threshold $(E_b/N_0)^* \approx 0.8216$ dB.

Regular SC-LDPC Codes vs. Irregular LDPC-BCs

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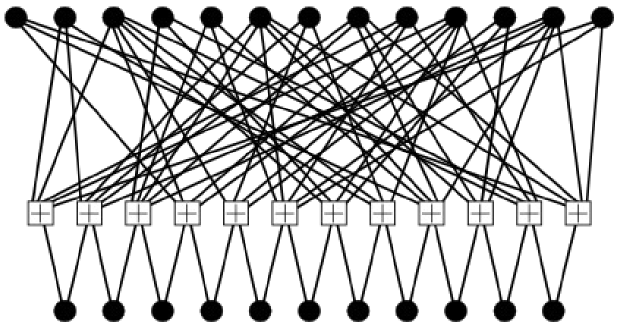

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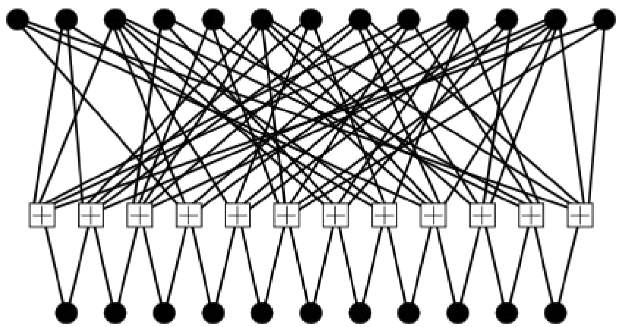

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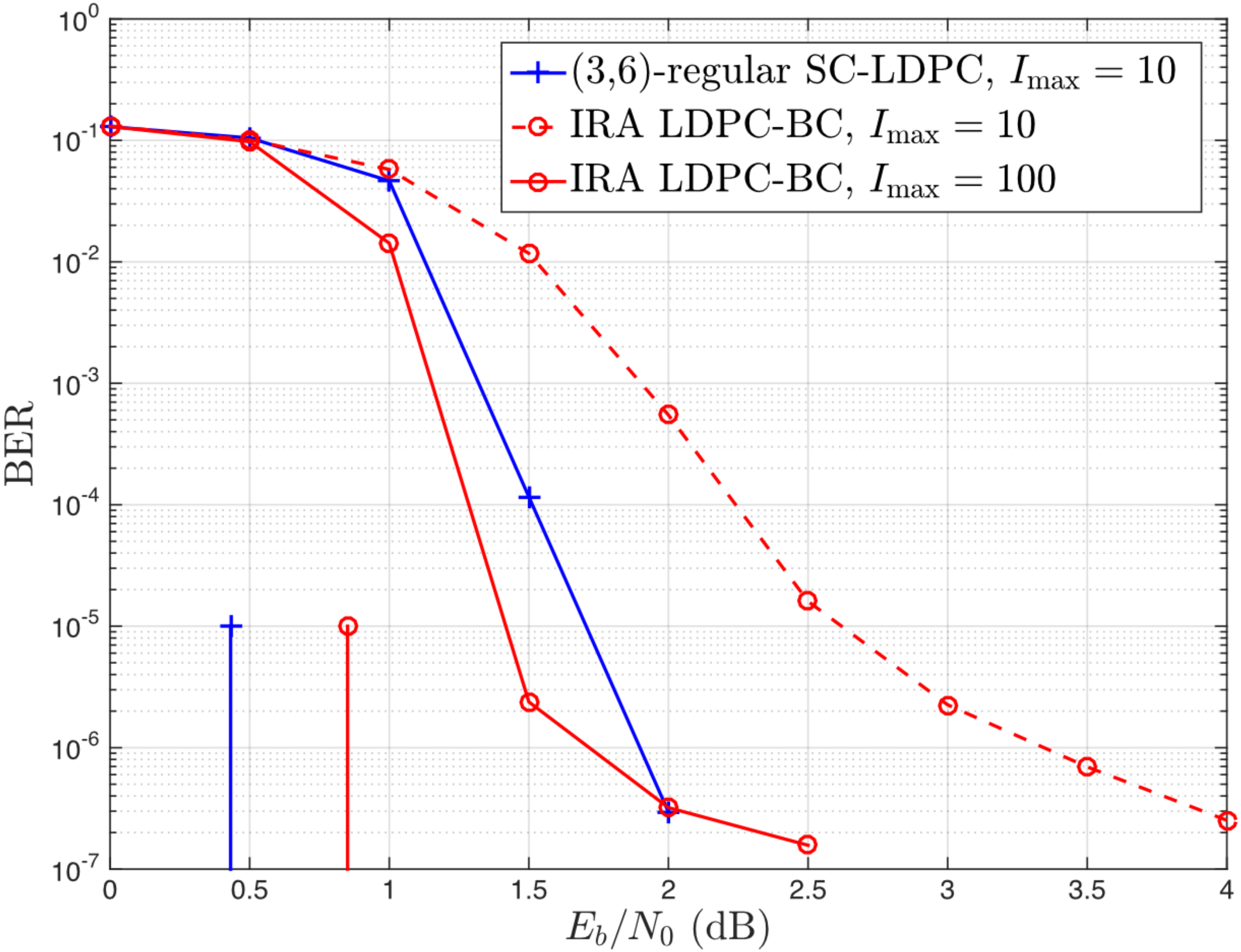
- For the SC-LDPC code, we choose W=6 and M=500 so that the latency of both codes is 6000 bits. (Since a code symbol is present in W=6 'windows', we allow fewer iterations per position for the SC-LDPC window decoder.)

Regular SC-LDPC Codes vs. Irregular LDPC-BCs



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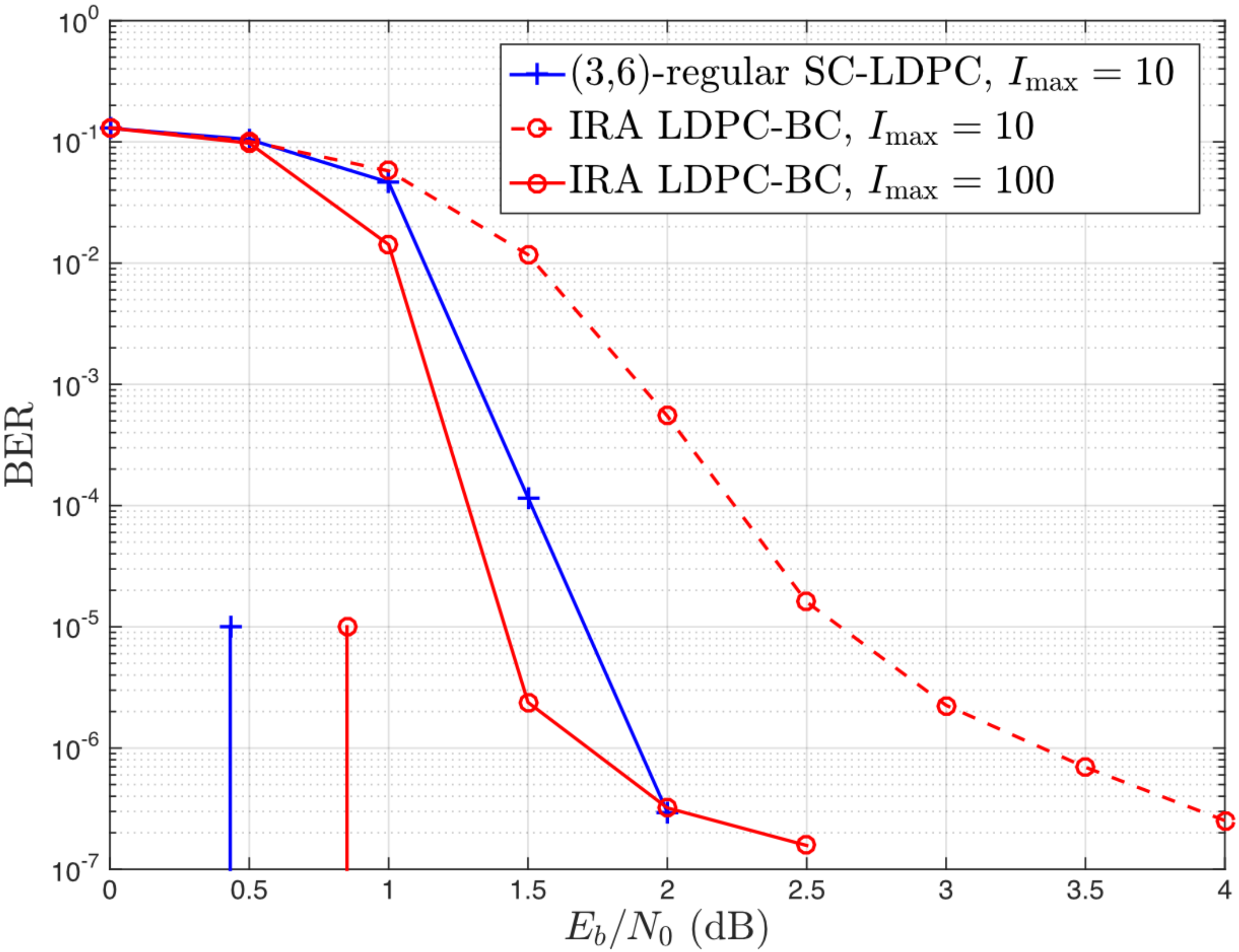

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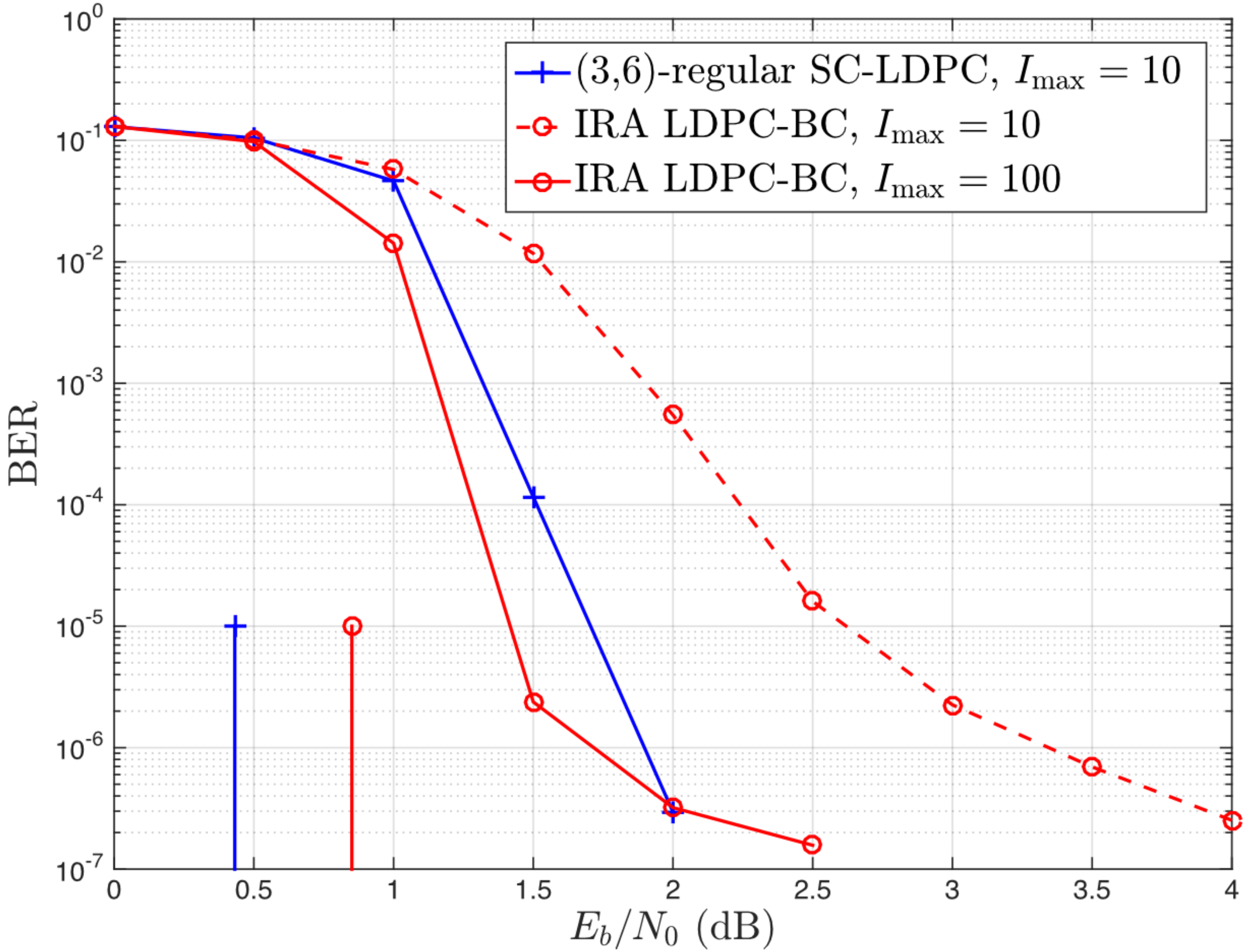
Regular SC-LDPC Codes vs. Irregular LDPC-BCs



- Gaps to threshold will reduce with increasing latency
- The asymptotically good regular SC-LDPC code shows no sign of an error floor
- The regular SC-LDPC code structure has implementation advantages

Implementation Aspects

- As a result of their capacity approaching performance and simple structure, regular SC-LDPC codes may be attractive for future coding standards. Several key features will require further investigation:
  - ➡ Hardware advantages of QC designs obtained by circulant liftings
  - ➡ Hardware advantages of the 'asymptotically-regular' structure
  - ➡ Design advantages of flexible frame length and flexible rate obtained by varying M, L, and/or puncturing

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  - ➡ Design advantages of flexible frame length and flexible rate obtained by varying M, L, and/or puncturing
- Of particular importance for applications requiring extremely low decoded bit error rates (*e.g.*, optical communication, data storage) is an investigation of error floor issues related to **stopping sets**, **trapping sets**, and **absorbing sets**.

Conclusions

University of Notre Dame

- Spatially coupled LDPC code ensembles achieve **threshold saturation**, i.e., their iterative decoding thresholds (for large L and M) approach the MAP decoding thresholds of the underlying LDPC block code ensembles.

- The threshold saturation and linear minimum distance growth properties of (J,K)-regular SC-LDPC codes combine the best asymptotic features of both regular and irregular LDPC-BCs.

- With window decoding, SC-LDPC codes also compare favorably to LDPC-BCs in the finite-length regime, providing flexible tradeoffs between BER performance, decoding latency, and decoding complexity.

- SC-LDPC codes can be punctured to achieve robustly good performance over a wide variety of code rates.